

# EPISTEMIC CLOSURE

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## 1. The Basic Idea

Do we know everything which is entailed by what we know? This would suggest the following principle:

If S knows that  $p$ , and if  $p$  entails  $q$ , then S knows that  $q$ .

(This would hold for all subjects S and all propositions  $p$  and  $q$ , of course; for the sake of simplicity we can skip this clause in the following.) In other words, knowledge is “closed under entailment.” This principle seems much too strong. It would only be true of logically omniscient subjects (who know all the logical consequences of what they know). For instance, everyone who knows certain mathematical axioms would thus also know all the mathematical theorems which follow from the axioms; if one does not know one of these theorems, then one also does not know the axioms.

A more restricted principle seems much more plausible in the case of human subjects:

(C1) If S knows that  $p$ , and if S knows that  $p$  entails  $q$ , then S knows that  $q$ .

In other words, knowledge is “closed under known entailment” (or “closed”; see for (C1) and for the principle above, Hintikka 1962: 30–38). If Mary knows that both Paul and Peter are at the party, then she also knows that Peter is at the party (given her knowledge that the latter follows from the former). If we were to deny that Mary knows that Peter is at the party, then we would be ready to deny that she knows that Peter and Paul are at the party (given her knowledge that the latter entails the former).

## 2. Importance: Skepticism

(C1) and similar principles play an important role in epistemology. Doesn't the possibility of inferential knowledge require the truth of some closure principle? And, apart from that, doesn't closure play an important role in discussions of epistemic skepticism? Here is the template of a typical skeptical argument crucial to the recent discussion about epistemic skepticism (with “ $\sim$ ” as the negation sign, “ $\rightarrow$ ” as the sign for material implication, “K” for “S knows that,” “s” for some skeptical proposition—like *I am a handless brain in the vat merely imagining that I have hands* (see Putnam 1981)—and “o” for some ordinary proposition—like *I have hands*):

- (1)  $\sim K \sim s$
- (2)  $(\sim K \sim s) \rightarrow (\sim Ko)$
- (C)  $\sim Ko$

I don't know I have hands because I don't know I am not a brain in a vat and if I don't know the latter, then I don't know the former. The crucial premise (2) follows from (C1) together with the relatively unproblematic assumption that  $K(o \rightarrow \sim s)$ .—Some anti-skeptics (called “Neo-Mooreans”) like to turn this kind of argument on its head, again relying on closure:

- (1')  $Ko$
- (2')  $(Ko) \rightarrow (K \sim s)$
- (C')  $K \sim s$ .

I know I am not a brain in a vat because I know that I have hands and if I know the latter then I also know the former. Some of those who are not convinced by the latter argument but do not want to give in to skepticism propose to give up (C1) and similar or related principles (see Dretske 2005a but cf. also Brueckner 1985). Whatever option one chooses, it seems that closure—whether in the form of (C1) or of some other principles (see below)—plays an important role for the discussion of at least some important forms of epistemic skepticism (but cf. also Brueckner 1994, Cohen 1998 and David and Warfield 2008).

### 3. Refinements

The closure principle (C1) requires some refinements and modifications. Suppose Frank knows that the temperature has just fallen below freezing point. Suppose he also knows that if the temperature is below freezing point, then driving can be dangerous. However, it seems perfectly possible that Frank does not “put two and two together” and thus does not know that driving can be dangerous now. Only if Frank makes the relevant inference, does he come to know that driving can be dangerous now. To accommodate such cases, the following modification seems appropriate:

- (c2) If S knows that  $p$ , knows that  $p$  entails  $q$ , and deduces from all this that  $q$ , then S knows that  $q$ .

Now, “ $p$ ” could stand for a complex proposition of the form “ $p$  and  $(p \rightarrow q)$ .” This gives us the case of “single-premise closure” which is at the centre of the current discussion about closure (see sec. 7 below for multi-premise closure):

- (c3) If S knows that  $p$ , and deduces  $q$  from  $p$ , then S knows that  $q$ .

Sometimes people propose to weaken (C1) in the following way rather than moving on to (C2) or (C3):

- (c1') If S knows that  $p$ , and if S knows that  $p$  entails  $q$ , then S is in a position to know that  $q$ .

However, being in a position to know is not the same as knowing; hence (C1') is too weak for our purposes; we would not be talking about knowledge closure any more but rather about being-in-a-position-to-know closure. And, apart from that, it is not clear at all what is required for being in a position to know; this idea seems a bit too unspecific to be of much help (see David and Warfield 2008: sec. IV).

Now, the deduction mentioned in (C3) needs to be a correct one and based on a general ability of the subject to draw such a correct inference. Let us say that in such a case the subject “competently” deduces  $q$  from  $p$ . If Frank makes an invalid inference even to a true conclusion, he might not come to know the conclusion. Also, if he just happens to make a correct inference when he usually fails to get these things right, he also might not come to know the conclusion. Hence, we had better replace (C3) by

(C4) If S knows that  $p$ , and competently deduces  $q$  from  $p$ , then S knows that  $q$ .

But what if Frank is the kind of person who simply cannot believe that it could ever be too dangerous for him to drive? Even though he deduces  $q$  from  $p$ , he does not come to believe that  $q$  (and he does not come to believe it on the basis of his deduction; see on this Harman 1986: 11–12). Let us modify (C4) then (see also Bogdan 1985; Hales 1995: 188–192):

(C5) If S knows that  $p$ , competently deduces  $q$  from  $p$ , and believes that  $q$ , then S knows that  $q$  (see, e.g., Forbes 1984: 43, 49–50 and Williamson 2000: 117 who add that S “thereby” comes to believe that  $q$ ; see also (T) below).

(C5) expresses not just a contingent fact but a necessity. Hence, we can add that “necessarily” (C5) is the case.—What if S ceases to know that  $p$  in the process of deducing  $q$  from it? Some (see, e.g., Hawthorne 2004: 29, 33) hold that we have to add a further condition: that S retains knowledge that  $p$  throughout the process of deduction.—Kvanvig (2006: 261–262) proposes a further qualification: that S learns of no undefeated defeater for  $q$  (e.g., someone extremely trustworthy just told me that not  $q$  and even though they are wrong there is no indication of that). Thus we get:

NECESSARILY: If S knows that  $p$ , competently deduces  $q$  from  $p$  while retaining knowledge that  $p$  throughout the inference, believes that  $q$  and does not learn of any undefeated defeater for  $q$ , then S knows that  $q$ .

There might be further conditions to be added (see David and Warfield, Ms.; David and Warfield 2008: secs. II–III; Warfield 2004) but we don’t need to get into a potentially endless discussion (resembling the post-Gettier discussion about *knowledge*); it is not clear whether one can indicate sufficient conditions for inferential knowledge that  $q$  (similar things hold of similar principles below). Anyway, the basic idea should be both clear and robust enough now. (C5) expresses the core of the idea of knowledge closure currently discussed. I will stick with (C5), also for the sake of simplicity and because nothing in the following depends on the added subtleties.

#### 4. Transmission

Closure principles remain silent about where S's knowledge that  $q$  comes from: S need not know  $q$  on the basis of deduction from  $p$ . Now, one might want to stress that S's knowledge that  $q$  can be derived from or based upon S's knowledge that  $p$  (where  $p$  differs from  $q$ ). This gives us a transmission principle:

- (T) If S knows that  $p$ , competently deduces  $q$  from  $p$ , and believes that  $q$ , then S knows or comes to know that  $q$  on the basis of that deduction (no matter whether  $q$  is also known independently from the knowledge that  $p$ ).

This is a transmission principle in the sense that it specifies the conditions under which knowledge can "transmit" by deduction from one proposition ( $p$ ) to another ( $q$ ). Closure principles like the ones above do not say anything about transmission as such. Unfortunately, closure and transmission principles are very often not, or not clearly, distinguished; sometimes it is very difficult to tell whether an author talks about closure or transmission of knowledge (for a clear distinction, see: Wright 1985: 438, fn. 1, 2000: 141; Davies 1998: 325–326, 2000: 393–394, 2003b: 108; McLaughlin 2003: 83; Dretske 2005a: 15, 16).

Very often transmission is explained in terms of transmission of warrant (where warrant is taken as whatever turns true belief into knowledge; but cf., e.g., Williamson 2000). This results in principles of the transmission of warrant across valid inference, like, e.g., the following one:

- (TW) If S has warrant for (believing that)  $p$ , competently deduces  $q$  from  $p$ , and believes that  $q$ , then S acquires (for the first time or not) warrant for (believing that)  $q$  on the basis of that deduction (and S can thus come to know that  $q$ ).

It does not matter here whether the subject believes or knows she has warrant or whether the subject believes or knows that the warrant supports the relevant proposition. The discussion about the transmission of warrant principles does not seem to depend on what position one takes on such issues.

If having warrant for a true belief is sufficient for turning that belief into knowledge, then every case of a true belief that  $q$  which meets a transmission principle like (TW) also meets a closure principle like (C5). The reverse, however, is not true (see above): A case can meet (C5) but not (TW) because the subject's knowledge that  $q$  might not be derived from or based upon their knowledge that  $p$ . I know that there is an apple in front of me and I also know that this entails that there is no fake apple in front of me; according to many, even if I know that there is no fake apple in front of me, I cannot know this on the basis of my knowledge that there is an apple in front of me (more on such cases below).

#### 5. Denials: Dretske and Nozick

The conviction that some closure (or transmission) principle is true is very widespread in contemporary philosophy. It seems very hard to deny that we can acquire new knowledge by making inferences from what we already know. Denying closure (or transmission)

seems to jeopardize the very idea of inferential knowledge. And how abominable does it sound to say, e.g., that “I know that I am sitting on a chair but I don’t know that I am not merely dreaming that I am sitting on a chair”?

However, there are dissenters. One of the first philosophers who can be read as denying closure is Ludwig Wittgenstein. In *On Certainty* (1969) he makes remarks which suggest that what we know is based on and presupposes things we don’t know (see also, more recently: Greene 2001: 67–71; Harman and Sherman 2004). If we read “based” and “presupposes” as involving some known entailment relation (between what we know and what we presuppose), then all this comes to a denial of closure. I know there is a tree in the yard but I don’t know there is an external world. Much more explicit is the denial in the case of some more recent epistemologists, particularly Robert Nozick and Fred Dretske. In Nozick’s case, the failure of closure is a consequence of his account of knowledge. His “Sensitivity” or “Tracking Account of Knowledge” has it that (with “iff” for “if and only if” and with “ $\Rightarrow$ ” as the sign for the subjunctive conditional, that is, for a conditional of the form “If  $p$  were the case, then  $q$  would be the case”)

S knows that  $p$  iff

- (a)  $p$ ,
- (b) S believes that  $p$ ,
- (c)  $(\sim p) \Rightarrow$  S does not believe that  $p$ , and
- (d)  $p \Rightarrow$  S believes that  $p$ .

S knows that there is a dog over there just in case the following conditions are met: S truly believes this (a, b); furthermore, if there were no dog over there, S would not believe it (c); finally, under different possible circumstances under which there is a dog over there, S would believe it (d). Sure, knowledge does not require that S’s belief “tracks” the truth even under the wildest possible circumstances (in the presence of artificial fake-dogs, etc.). Therefore, the subjunctive conditionals (c) and (d) have to be evaluated not with respect to all possible worlds but only to “close” ones—to those which resemble the actual circumstances very much. Furthermore, one has to take into account the way in which the belief was formed (see Nozick 1981: 172ff.). Such details left aside, it becomes clear quickly that this account is not compatible with any of the closure (or transmission) principles mentioned so far (see Nozick 1981: 206, 227–240). Consider this case. Bob knows that he has hands because he has hands, he believes he has hands, in all close possible worlds in which he has hands he believes this, and in all close possible worlds in which he lacks hands (e.g., because of an accident) he does not believe he has hands. Suppose Bob also knows that if he has hands then he is not a handless brain in a vat. Does he or can he also know that he is not a handless brain in a vat? According to Nozick’s condition (c), if Bob were a handless brain in a vat, he would have to not believe that he is not a handless brain in a vat. However, given what it means to be a brain in a vat, he would believe that he is not a handless brain in a vat. Hence, Bob does not meet Nozick’s condition (c) and thus does not know that he is not a handless brain in a vat. The problem for closure is now obvious: According to Nozick, one can know something even without knowing something else which one knows follows from what one knows. Closure fails (where closure is understood along the lines of (C5) and similar principles). Many philosophers hold the closure principle so dear that they hold the failure of closure against Nozick’s account of knowledge. Nozick went the other way and proposed to give up closure principles like (C5). One advantage in the

case of skeptical arguments is that Nozick can thus accommodate both our view that we know a lot of ordinary propositions and the skeptical insight that we don't know that we're not brains in a vat. Other modal accounts, of knowledge, like the safety view (see, e.g., Sosa 1999), also have problems with closure (see the dachshund example in Goldman 1983: 84, which has an echo in a well-known similar example by Saul Kripke; Sosa (1999: 292–294) concedes that there is a problem).

Dretske's denial of ordinary closure principles is at least partly motivated by reasons very similar to Nozick's (see Dretske 2003: 113, 2005a: 19–20, 2005b: 43–44). However, it also seems triggered by the discussion of cases. Here is one of Dretske's most well-known examples (see Dretske 1970, 1982; cf. critically Klein 1981: 29–33 and Vogel 1990: 13–15; see, for other examples, Audi 1998: 169 and the debate on it in: Feldman 1995 and Audi 1995; see also Hookway 1989/90: 9–10; Maitzen 1998; Salmon 1989: sec.VI). Suppose Fred is in the zoo and standing in front of the zebra cage. Does he know that he is looking at zebras? There seems to be no reason to deny this. But does he know that he is not looking at cleverly disguised mules (painted as zebras)? It seems clear that he does not know that. However, the latter proposition can be easily known by Fred to be entailed by the former one. Hence, closure fails. Dretske also applies this to contemporary skeptical arguments: One can know that there is a cup in front of one even if one does not know what is known to be entailed by it, e.g., that one is not merely dreaming that there is a cup in front of oneself. Like Nozick, Dretske can thus make peace between the skeptic and the anti-skeptic (see Dretske 2005a: 18, 23). One can also explain all this in terms of Dretske's "relevant alternatives theory of knowledge" (Dretske 1981, 2005a: 19; McGinn 1984: 544; Stine 1976; Lewis 1996; Heller 1999; for Dretske's view on information and closure see Dretske 2003: 115, 2004: 176–177).

It should be stressed that both Dretske and Nozick seem to talk not just about closure but about transmission principles, too (see Nozick 1981: 205–206). More importantly, I should stress that, strictly speaking, neither Dretske nor Nozick opt for an unrestricted denial of any principle of closure (not just (C5)). Dretske accepts closure in cases of (known) conjunction-elimination (Dretske 1970; but cf. Hawthorne 2005: 31–32) while Nozick denies it in those cases (as well as for universal instantiation; see Nozick 1981: 227–229); Nozick accepts closure for cases of (known) equivalence, existential generalization, conjunction-introduction or disjunction-introduction (see Nozick 1981: 229, 230, 236; cf. p. 231 for a restriction of closure rather than its denial). Furthermore, Dretske has later argued in a way which seems to explicitly suggest a restriction of closure principles rather than their abolishment (see Dretske 2005a: 16, 17 or 2003 where he argues that closure breaks down when "q" stands for a "heavyweight" or "limiting proposition"). And Roush (2005) has defended a closure-compatible version of Nozick's theory. Looking at such later modifications, one should conclude that the real controversy is much more about the proper formulation of a closure (or transmission) principle than about the acceptance of any such principle (see Goldman 2008: 478–479; but also Becker 2007: 113–128, who, supporting Nozick, recommends giving up closure, or Adams and Clarke 2005 who defend the tracking account against common counterexamples; see also Luper-Foy 1987).

## 6. Problems: Lotteries and Easy Knowledge

There are also some serious problems which anyone who wants to defend some closure (or transmission) principle would have to address. The apparent incompatibility of

ordinary knowledge claims, closure and the impossibility to know one is not in a skeptical scenario has already been mentioned above. Let us now look at a problem first brought up by Gilbert Harman (see Harman 1973: 161; see also Hawthorne 2004 and Vogel 1990: 15–20). Suppose you believe that you will never be a millionaire. Suppose that this is in fact true and you know it. Now, you also know that if you will never be a millionaire, then you will never win the millionaires' lottery. But how can you know that you won't win that lottery? If we assume—with many contemporary epistemologists—that one cannot know that one won't win a lottery (given normal circumstances, like having bought a ticket for a fair and unrigged lottery, etc.), then it seems very hard to stick with the claim to know that one will never be rich (given closure). This problem easily generalizes to all pairs of propositions such that one proposition is an ordinary proposition we would tend to claim to know and the other proposition is a “lottery” proposition entailed by the first one; a lottery proposition is a proposition which has a high probability of being true but which we would typically not tend to claim to know (for more on the notion of a lottery proposition see Vogel 1990: 16–17). Here is another example, not involving a literal lottery. Frank knows that his children are playing in the garden but he does not know that his children have not been kidnapped and replaced by actors who pretend convincingly to be his children. Since it is hard to think of any ordinary proposition which does not entail some lottery proposition and since such an entailment can easily be known by the subject, our problem is hard to contain and concerns all kinds of propositions we ordinarily think we know. Three main responses are on offer in the current debate. First, one could accept closure as well as the idea that one cannot know a lottery proposition (both plausible); but then one also has to accept the resulting widespread skepticism (not plausible to many). Second, one could accept closure as well as our ordinary knowledge claims (both plausible); but then one would also have to be ready to accept the possibility of knowledge concerning lottery propositions (not plausible to many). Finally, one could accept both skepticism concerning lottery propositions as well as our ordinary knowledge claims (both plausible); but then one would have to reject closure (cf. from the recent debate: Hawthorne 2004). An analogous problem arises, of course, for transmission principles.

Another problem is the problem of “bootstrapping” and “easy knowledge” (see Cohen 2002; Sosa 2009: chs. 4, 5, 9, 10). My speedometer tells me that I am driving at 40 mph. Suppose I am traveling at that speed. It seems hard to deny that I can thus come to know that I am traveling at 40 mph. I can also come to know that the speedometer indicates a speed of 40 mph (by looking at it). From both I can infer that the speedometer is indicating the speed correctly. If I repeat this little exercise many times, I can infer that the speedometer is working reliably. However, these conclusions do not seem to constitute knowledge. But they follow from other things I know and the entailment is also known. Here is another example of the same kind. I look in front of me and thus come to know that there is an apple in front of me. I also know a real apple is not a fake apple made to look like an apple. I can conclude but, it seems, not come to know in this way that there is no such fake apple in front of me. Knowledge cannot be acquired that easily. What has gone wrong? And what ought we to say about such cases? Should we deny knowledge of one of the two premises? Should we allow for bootstrapping and easy knowledge? Or should we deny closure? All three options seem implausible. Again, a parallel problem arises for transmission principles.

One way out would be to restrict the closure principle in a certain way. Here is a rough idea. What is common to such cases of bootstrapping and easy knowledge is that

the subject is granted knowledge of the premises but only insofar as the subject is also granted certain assumptions or presuppositions even if they do not constitute knowledge from the outset: that the speedometer is working, that the subject's vision is fine, etc. If we were to not grant the subject these assumptions and presuppositions—if, for instance, we saw good reasons to doubt that everything is fine with the speedometer or the eyes of the subject—then we would not attribute knowledge of the premises to the subject. This suggests the following modification of (C5):

- (C6) If S knows that  $p$ , competently deduces  $q$  from  $p$ , and believes that  $q$ , then S knows that  $q$ —but not if  $q$  is both antecedently unknown by S but taken for granted and presupposed by S's belief and knowledge that  $p$  (see Barke 2002: 164–166 and even Dretske 1970: 1014 as well as Nozick 1981: 239–240).

Even if a closure principle like (C6) holds in certain cases, transmission might still fail. In any case, it is good to have a restricted transmission principle which takes care of such potential failures of transmission. Here is a proposal (for the case of transmission of warrant):

- (TW\*) If S has warrant for (believing that)  $p$ , competently deduces  $q$  from  $p$ , and believes that  $q$ , then S acquires (for the first time or not) warrant for (believing that)  $q$  on the basis of that deduction (and may thus come to know that  $q$ )—but not if the having of the warrant for (believing that)  $p$  depends on antecedent reliance on  $q$  (for more along such lines, see Wright 1985: 432–438, 2000, 2002, 2003, 2004, 2007; Davies 1998: 351–355, 2000: 402–412, 2003a: 30–45, 2003b: 122–130, but cf. 2004; see further McLaughlin 2003: 91, 84–91; Brown 2004; Pryor 2004 and forthcoming; Okasha 2004; Silins 2005; Nozick 1981: 239–240; Olin 2005: 237–238, 243).

We can leave the question open whether principles like (C6) or (TW\*) can help us deal with the problem of easy knowledge, bootstrapping or with the lottery problem (see Hawthorne 2004). There is some hope but we cannot go into that here.

## 7. Further Problems: Multi-Premise Closure and Probability

So far we have only considered cases in which the subject deduces one proposition from another proposition. But what about a case like the following? You're throwing a party and have invited all of your 100 friends. Alfonsina has accepted the invitation and promised to come. You believe her because you know her to be extremely reliable; as a matter of fact, she will indeed show up. It seems uncontroversial to say that you can thus come to know that she will come to your party. Similar things are true of Bernie, Claire and all your other friends. You know 100 premises where each premise says of one of your friends (a different one each time, of course) that they will come to the party: You know that  $p1$ , that  $p2$ , . . . that  $p100$ . You also know about conjunction-introduction. You thus make the inference to the conjunction and conclude that all of your 100 friends will attend the party. But even if they all do this hardly seems to be knowable by you. Again, we have a problem: the knowledge of the premises,



some closure principle and the lack of knowledge of the conclusion are incompatible with each other.

What creates the problem? Here is an idea. For each individual proposition there is a very high though not maximal (subjective) probability that it is true. Let us assume the probability is .99 in each case. It seems plausible that such a proposition can be known to be true despite the element of fallibility. Let us further assume that all these propositions are probabilistically independent from each other. The probability of the conjunction of all these propositions is then much smaller than .99 (namely less than .37). It is hard to believe that such a relatively unlikely proposition can still be known. However, it follows from all of the highly likely individual propositions, given plausible logical principles (there are obvious parallels here with Kyburg's lottery paradox (Kyburg 1961: 197) as well as with the preface paradox (Makinson 1965; Olin 2005)).

What should one do about this? Skepticism with respect to the individual premises appears too high a price to pay. Even less plausible is it to claim that one can come to know the rather unlikely conclusion. What then about denying multi-premise closure? This principle might seem much more plausible anyway than it ought to. The plausibility of closure principles seems to depend on using simple cases with just one premise; it seems to fade away as soon as we consider cases with more than one premise (see Hawthorne 2004: *passim*). (A very thorny problem comes with the question of how one should individuate premises: Is it always clear how many premises the subject used? Aren't there cases where one could count premises in more than one way, with different results?)

Perhaps we should then just give up the idea—bitter as it might seem—that there is more than single-premise closure to have (but cf. Stine 1976: 251 and Lasonen-Aarnio 2008). Neither the multi-premise parallel of the simple closure principle (C1),

(MC1) If S knows that  $p_1, \dots$ , and that  $p_n$ , and if S knows that  $p_1, \dots$ , and  $p_n$  together entail  $q$ , then S knows that  $q$ ,

nor the multi-premise parallel of the more complex closure principle (C6),

(MC6) If S knows that  $p_1, \dots$ , and that  $p_n$ , competently deduces  $q$  from  $p_1, \dots$ , and  $p_n$ , and believes that  $q$ , then S knows that  $q$ —but not if  $q$  is both antecedently unknown by S but taken for granted and presupposed by S's belief and knowledge that  $p_1$  or  $\dots$  by S's belief and knowledge that  $p_n$ ,

are true. The same holds, *mutatis mutandis*, for transmission principles (see also Olin 2005).

Not all acceptable inferences are deductive. Some inferences are “probabilistic” in the sense that the truth of the conclusion is not “guaranteed” by the truth of the premises but made probable by the premises. The probability of such a conclusion might not be extremely high but it can still be high enough for knowledge (again: if a probability less than 1 is compatible with knowledge). Here is an example: Ann's car is parked in front of her flat and she is usually at home when her car is there; hence, Ann is at home. If one knows the premise(s), then one should be able to come to know the conclusion—if it is true—by such an inference. This suggests probabilistic versions of our closure principles. Here is a principle parallel to (C1):

- (P1) If S knows that  $p$ , and if S knows that  $p$  makes  $q$  probable (enough), and if  $q$  is true, then S knows that  $q$ .

And here is the principle parallel to (C6):

- (P6) If S knows that  $p$ , competently makes a probabilistic inference to  $q$  from  $p$ , believes that  $q$ , and if  $q$  is true, then S knows that  $q$ —but not if  $q$  is both antecedently unknown by S but taken for granted and presupposed by S's belief and knowledge that  $p$ .

There will be similar transmission principles. But again, for the kinds of reasons explained above, one should not expect there to be true principles of multi-premise probabilistic closure.

## 8. Conclusion

The idea of closure of knowledge under known entailment is of great importance for epistemology (we had to leave closure principles for epistemic justification and other relevant notions aside here). There seems to be very widespread agreement—and not without good reason—that some closure principle has to be true. What looks like a contemporary debate between a majority of defenders of closure and a minority of deniers of closure is to a large degree (though not completely) a debate about the right kind of closure principle. It is plausible to assume that several whistles and bells have to be added to the simple closure principle (C1). Furthermore, there is not only a plausible closure principle for the case of deductive inference but also for the case of probabilistic inference. It is much harder to see how a principle of multi-premise closure could be true. There are also problems for principles of single-premise closure awaiting a convincing solution, such as Harman's problem or the problem of bootstrapping and easy knowledge. The related discussion about transmission of failure can give useful impulses to the debate about closure.

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