SSX scalings

In the spirit of my last note on the dimensionless Ohms law, here's a set of scalings for various parameters of interest for SSX. The idea is to have a set of formulae for quick calculations. These are all worth checking on your own. Let's set some nominal values for SSX:

$$\ell_{0.1m} \equiv 0.1 \ m$$
$$B_{0.1T} \equiv 0.1 \ T$$
$$T_{10} \equiv 10 \ eV$$

 $n_{14} \equiv 10^{14} \ cm^{-3}$

Using these, we can write down a few simple formulae:

$$\rho_{i} = 0.32 \ cm \frac{T_{10}^{1/2}}{B_{0.1T}}$$

$$\rho_{e} = 0.075 \ mm \frac{T_{10}^{1/2}}{B_{0.1T}}$$

$$\lambda_{D} = 2.35 \ \mu m \frac{T_{10}^{1/2}}{n_{14}^{1/2}}$$

$$\beta \equiv \frac{W_{kin}}{W_{mag}} = 0.03 \frac{n_{14}T_{10}}{B_{0.1T}^{2}}$$

$$N_{particles} \approx 10^{19} n_{14}$$

$$W_{mag} \approx 0.4 \ kJ \ B_{0.1T}^{2}$$

$$f_{ci} = 1.52 \ MHz \ B_{0.1T}$$

$$f_{ce} = 2.8 \ GHz \ B_{0.1T}$$

$$f_{pe} = 90 \ GHz \ n_{14}^{1/2}$$

What this means is that typically $\rho_i \cong 0.32 \ cm$ in SSX. If T_i is $4 \times$ larger, ie. $T_i = 40 \ eV$, then ρ_i is $2 \times$ larger, ie 0.64 cm. Note that ρ_i also scales like $\sqrt{M_{ion}}$ so the carbon ions we're measuring have a gyroradius $\sqrt{12} = 3.5 \times$ larger than protons at the same temperature. A typical DeBye length in SSX is a few microns and higher densities than 10^{14} only make it smaller.

$$v_A = 22 \frac{cm}{\mu s} \frac{B_{0.1T}}{n_{14}^{1/2}}$$

where $22 \frac{cm}{\mu s} = 220 \frac{km}{s}$
$$\tau_A = \frac{\ell}{v_A} = 0.46 \ \mu s \frac{\ell_{0.1m} n_{14}^{1/2}}{B_{0.1T}}$$

$$v_i = \sqrt{\beta} v_A$$

$$\delta_i = \frac{c}{\omega_{pi}} = \frac{2.3 \ cm}{\sqrt{n_{14}}}$$

$$\delta_e = \frac{c}{\omega_{pe}} = \frac{0.53 \ mm}{\sqrt{n_{14}}}$$

Now, if we invoke classical Spitzer resistivity, which is based on Coulomb collisions between ions and electrons, we have:

$$\eta = 5.15 \times 10^{-5} \frac{Z \ln \lambda}{T_e^{3/2}} \Omega m$$

where $ln\lambda$ is about 10 for SSX, Z is about unity, and T_e is expressed in eV. Plugging in the values for SSX at 10 eV we get a scaled resistivity:

$$\eta_{10}^{SSX} = 1.6 \times 10^{-5} T_{10}^{-3/2} \Omega m$$

Using this, we can calculate all the SSX parameters that depend on resistivity.

$$\tau_{res} \equiv \frac{\mu_0 \ell^2}{\eta} = 770 \ \mu s \ \ell_{0.1m}^2 T_{10}^{3/2}$$
$$S \equiv \frac{\tau_{res}}{\tau_A} = 1674 \ \frac{\ell_{0.1m} T_{10}^{3/2} B_{0.1T}}{n_{14}^{1/2}}$$
$$R_m = 76 \ v_{cm/\mu s} \ \ell_{0.1m} T_{10}^{3/2}$$

$$\tau_{whistler} = \frac{\ell^2}{\delta_e^2 \omega_{ce}} = 2 \ \mu s \ \frac{n_{14}}{B_{0.1T}}$$

Stuff you can do if you have these formulae memorized include quickly comparing our parameters to others. Our $\delta_i = 2.3 \ cm$. The magnetosphere plasma density is 10 $\ cm^{-3}$ or 10^{13} smaller than ours. Since $\sqrt{10^{13}} \cong 3 \times 10^6$, the magnetosphere ion inertial scale (important for reconnection there) must be 3×10^6 bigger than ours or about $10^7 \ cm$ or 100 km (which is right). Spacecraft measurements (Polar and Cluster, see Mozer PRL) show a reconnection layer in the magnetosphere about that scale.