Diffusivity
(connection between δ_e and η)

Collisions: The drag force on an electron due to collisions with ions comes from:

$$\mathbf{F}_{drag} = \nu_{ei} m_e \mathbf{u}_{rel} = e\mathbf{E}$$

This is just saying that the electric force is being balanced by collisions.

The collision frequency has the form:

$$\nu_{ei} = \frac{n_i Z^2 e^4 ln\lambda}{4\pi\epsilon_0^2 m^2 v^3}$$

where the v^{-3} dependence is evident (see earlier notes).

Now we remember Ohm's law: $\mathbf{E} = \eta\mathbf{J}$ and $\mathbf{J} = ne\mathbf{u}_{rel}$ so (combining the expressions for E):

$$\mathbf{E} = \eta ne\mathbf{u}_{rel} = \nu_{ei} m_e \mathbf{u}_{rel}/e$$

so we have an expression for the Spitzer resitivity:

$$\eta = \frac{m_e \nu_{ei}}{ne^2}$$

Electron inertia: The electron plasma frequency (in MKS) is (see earlier notes):

$$\omega_{pe}^2 = \frac{ne^2}{\epsilon_0 m_e}$$

The so-called electron inertial length is defined: $\delta_e \equiv \frac{c}{\omega_{pe}}$ We can construct a diffusivity (with dimensions $length^2/time$) using the collision frequency: $\delta_e^2 \nu_{ei}$. We can construct a fluid resisitive diffusivity from the Spitzer formula above: η/μ_0. Comparing these two, we find that they are the same thing!

$$\eta/\mu_0 = \frac{m_e \nu_{ei}}{ne^2 \mu_0} = \delta_e^2 \nu_{ei} = \frac{c^2 \epsilon_0 m_e \nu_{ei}}{ne^2}$$

This is interesting. It says that the MHD fluid resistivity can be thought of as electron diffusion at the electron inertial scale.