Turbulence and fluctuation studies in the SSX plasma wind tunnel

K. Flanagan, T. Gray, M. R. Brown

Swarthmore College, Swarthmore, Pennsylvania 19081

Introduction

Plasma physics is a diverse field of study with experiments ranging from large tokamaks focused on fusion research to spacecraft making measurements of the solar wind. The Swarthmore Spheromak Experiment (SSX) investigates fundamental plasma physics topics such as magnetic reconnection and turbulence. Currently, SSX is fitted with a 13:1 copper flux conserver that serves as a plasma wind tunnel. In this configuration, plasma turbulence might be expected. Motivated by recent publications on statistical methods used to measure turbulence in space plasmas [1,2], auto-correlation techniques were applied to data from magnetic probes. Additionally, spectral analysis on magnetic probes and a He-Ne interferometer showed fluctuations during reconnection events around the ion cyclotron frequency of the SSX plasma.

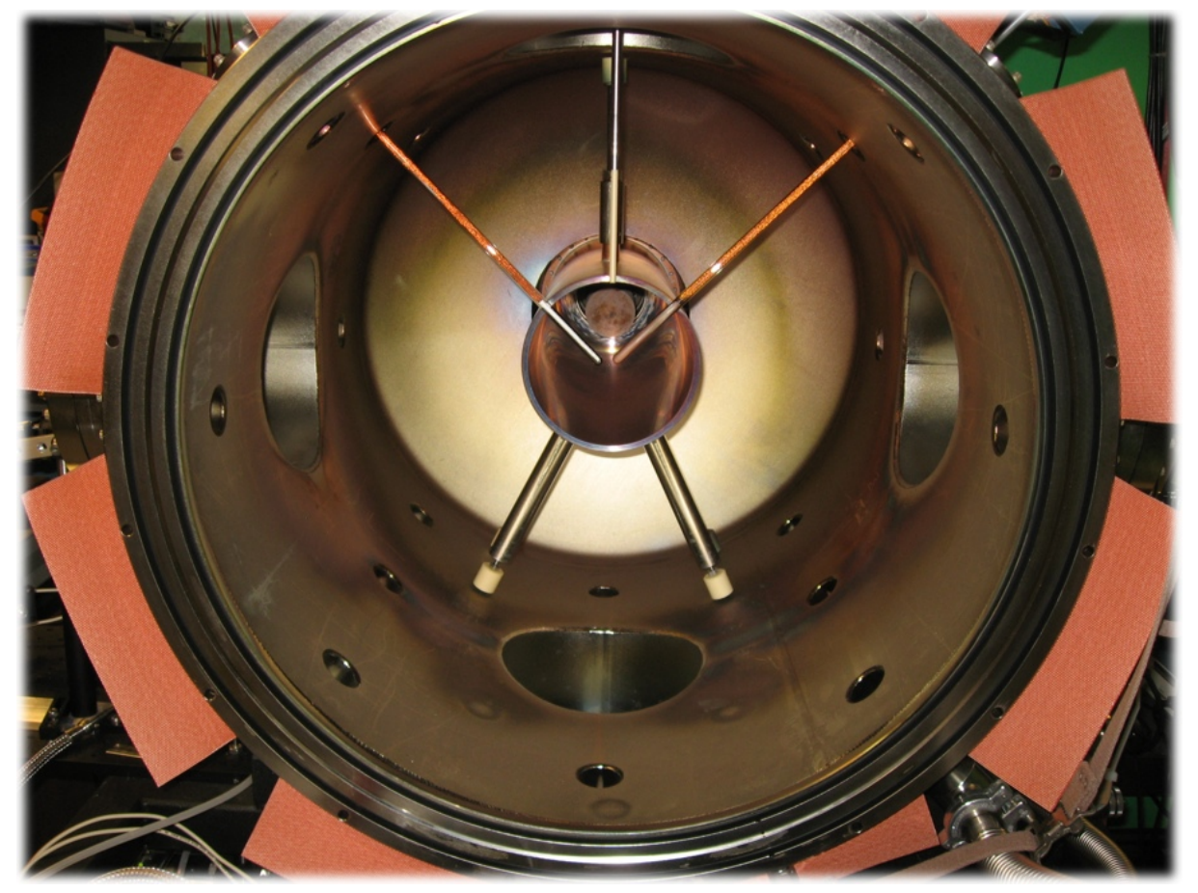


Figure. 1 SSX vessel opened at one end with half of the 13:1 flux conserver shown along with both radial magnetic probes; the radius of the flux conserver is ~7.5 cm

Theory

The spatial auto-correlation of a signal shows the similarity of the signal with itself as a function of separation. This is done by slipping the signal by a lag length and then convolving the slipped and original signals. Formally the spatial auto-correlation is:

$$A(z) = \langle B(r)\,B(r+z)\rangle \tag{1}$$

where $B(r)$ is the original signal and z is the lag length. The auto-correlation of a signal is related to its Fourier transform by the Wiener-Khinchin theorem:

$$\tilde{A}(k) = \tilde{B}^{*}(k)\tilde{B}(k) \tag{2}$$

where $\tilde{f}(k)$ is the Fourier transform of a function. Since fast Fourier transforms are easily computed for discrete data sets, the Wiener-Khinchin theorem is utilized in actually carrying out the auto-correlation of a signal in this analysis.

The auto-correlation function is expected to decay at large lag lengths since signals in plasma are not typically uniform over large distances. Fitting the auto-correlation functions to an exponential decay gives a characteristic length over which the signal can be considered highly correlated. The fitting function for a normalized auto-correlation signal is given by[1]:

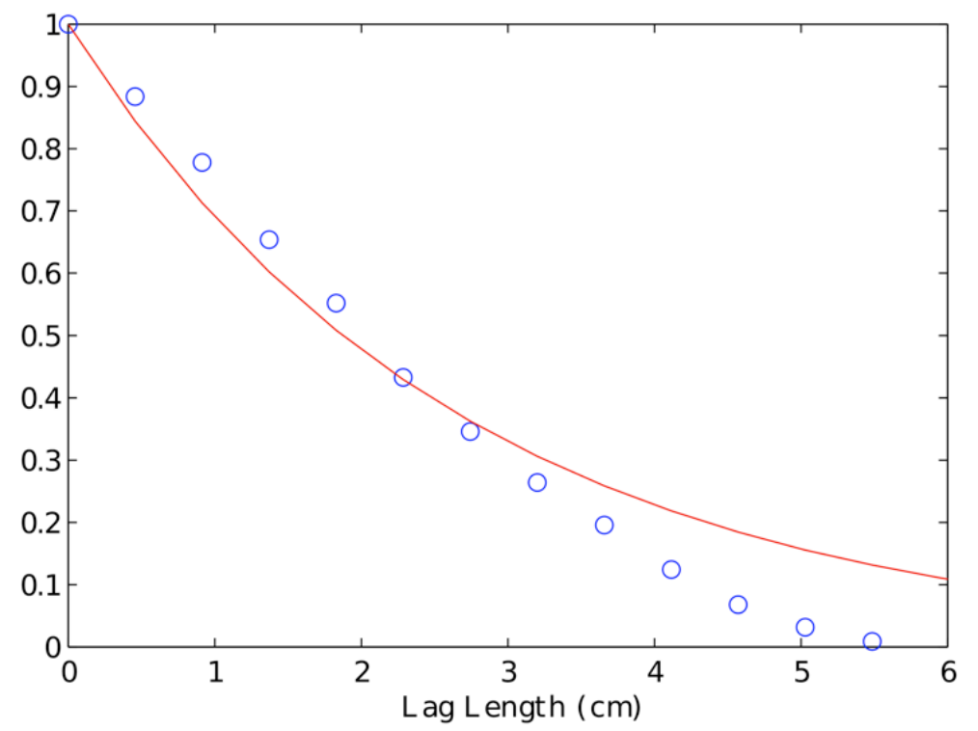


Figure. 2 A plot of the auto-correlation from a magnetic probe signal

$$A(z) \approx e^{-z/l_c} \tag{3}$$

where l_c is the characteristic length fitting parameter.

Magnetic probe configuration

The magnetic probes are composed of loops of magnet wire around a plastic rod sheathed in a quartz tube. The axial probe consists of loops in two orientations (x and y) at 19 positions spaced ~3.8 *cm* apart. The two radial probes each have 16 positions with three orientations (r, θ, and z) spaced ~0.46 *cm* apart (Fig. 3). The radial probes were placed at the midplane and set 90° apart.

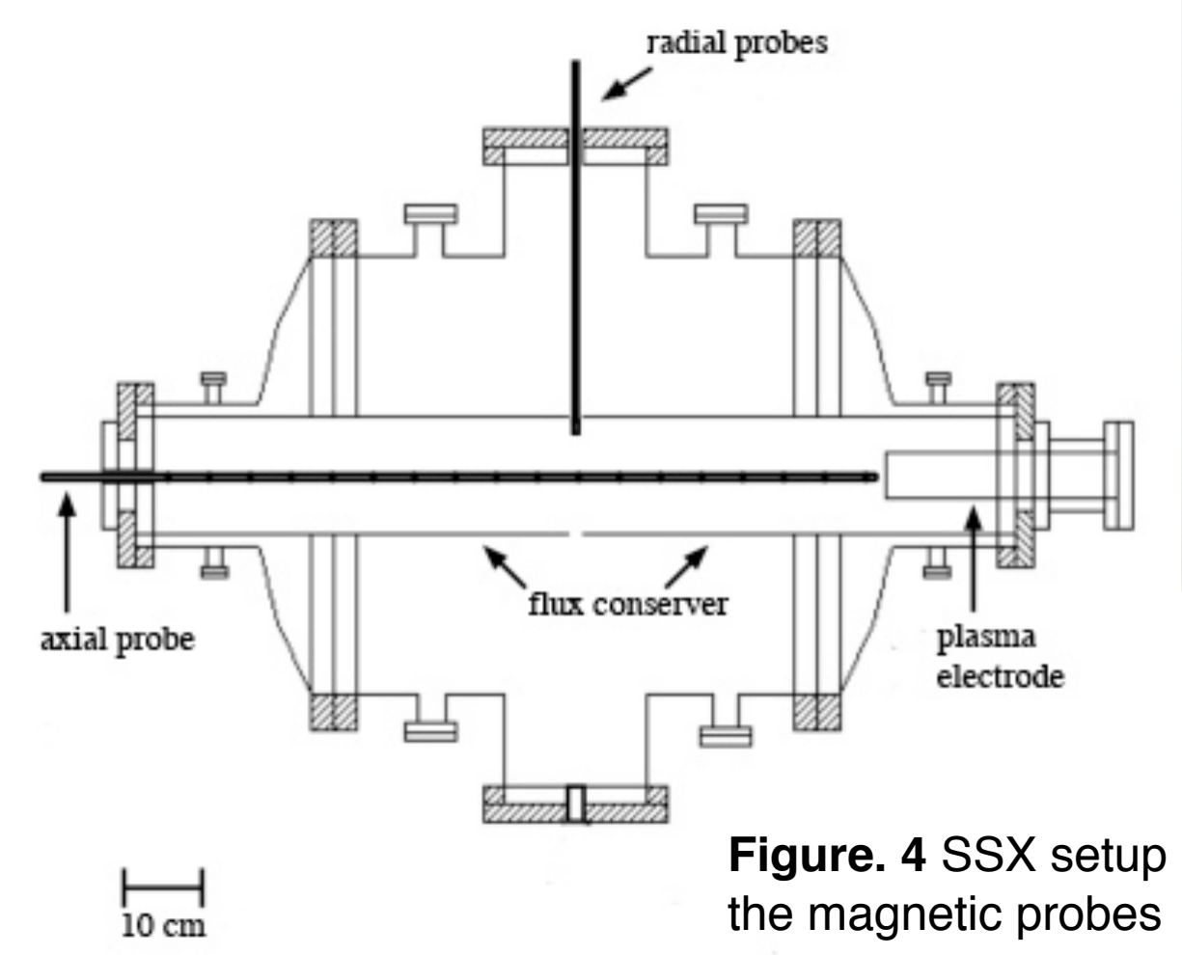


Figure. 4 SSX setup showing the magnetic probes

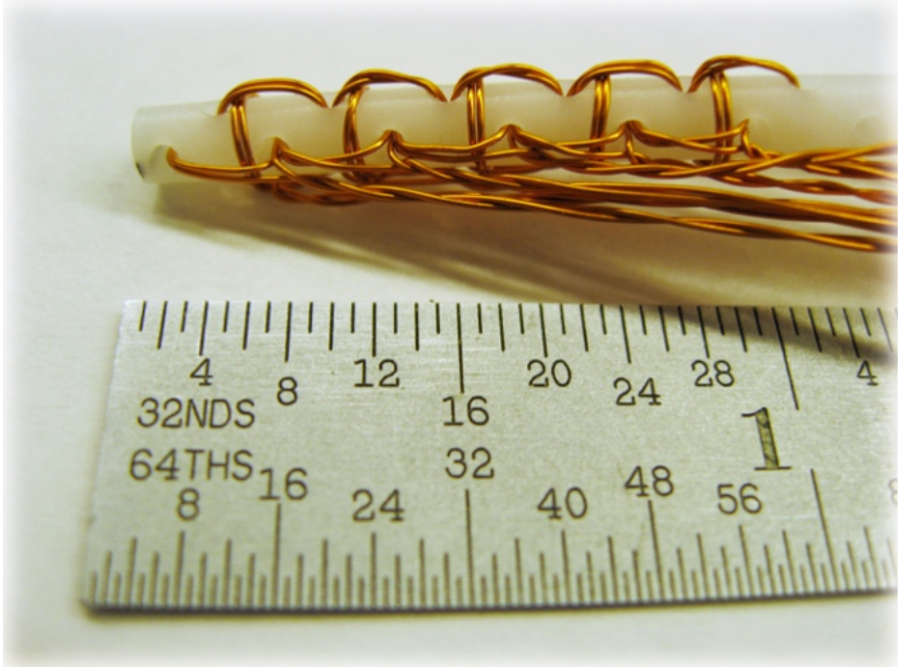


Figure. 3 One of the radial probes during construction

The signals from the magnetics probes were digitized at 65 *MHz* with 14 bit resolution. In order to take advantage of this bandwidth, the signals were not hardware integrated.

Axial probe measurements

The auto-correlation of the axial probe magnetic field signal shows a spatial periodicity across the probe later in time.

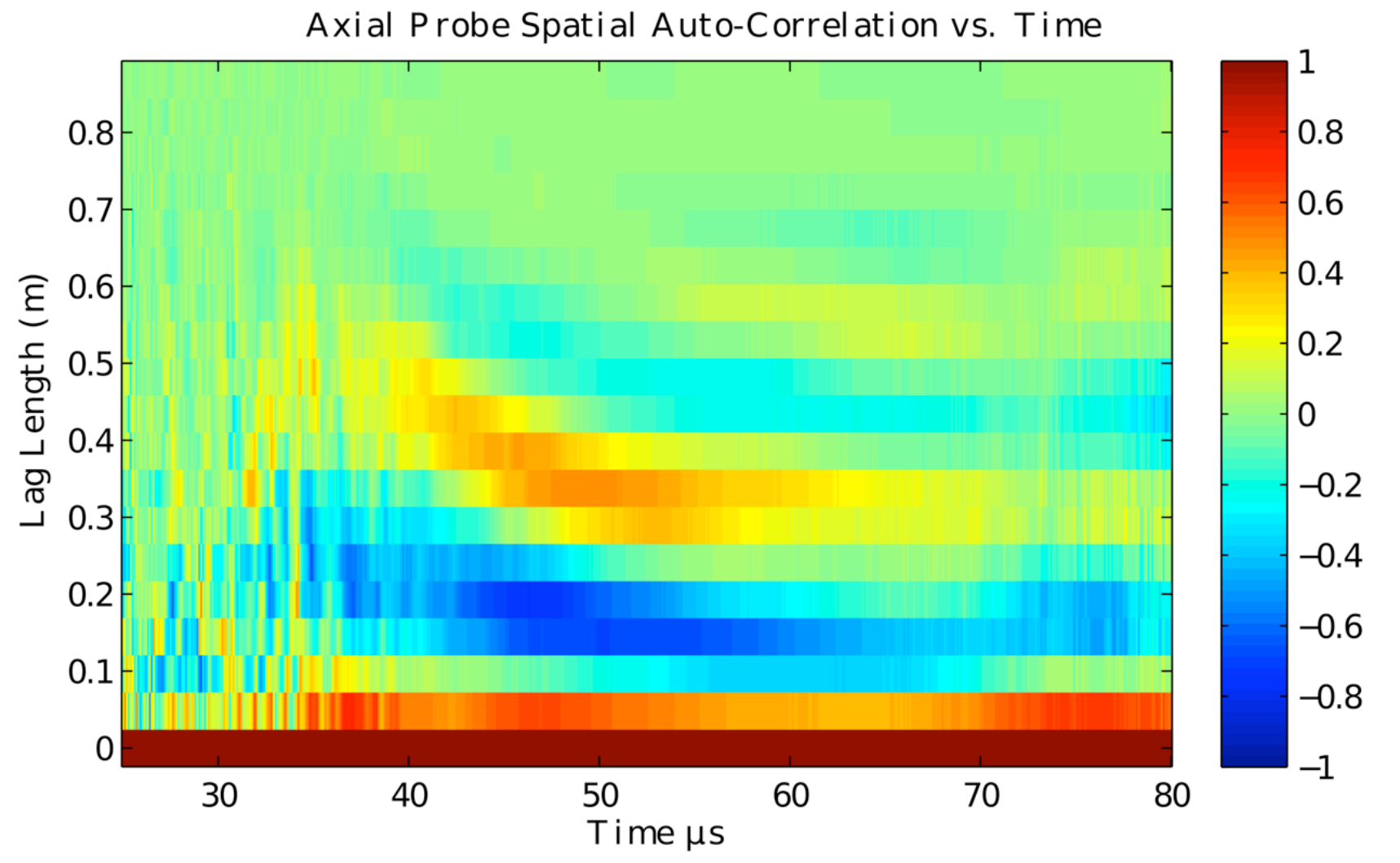


Figure. 5 Axial probe auto-correlation vs. time plot showing the ~40 cm feature of the relaxed taylor state emerging after a period of turbulent activity

This figure was produced by taking the normalized spatial auto-correlation function across the entire probe (Fig. 2) at each time step. The most notable feature of this plot is the periodic correlation and de-correlation at ~40 *cm*. The result is consistent with previous SSX results that show a relaxation of the spheromak into a twisted Taylor state.

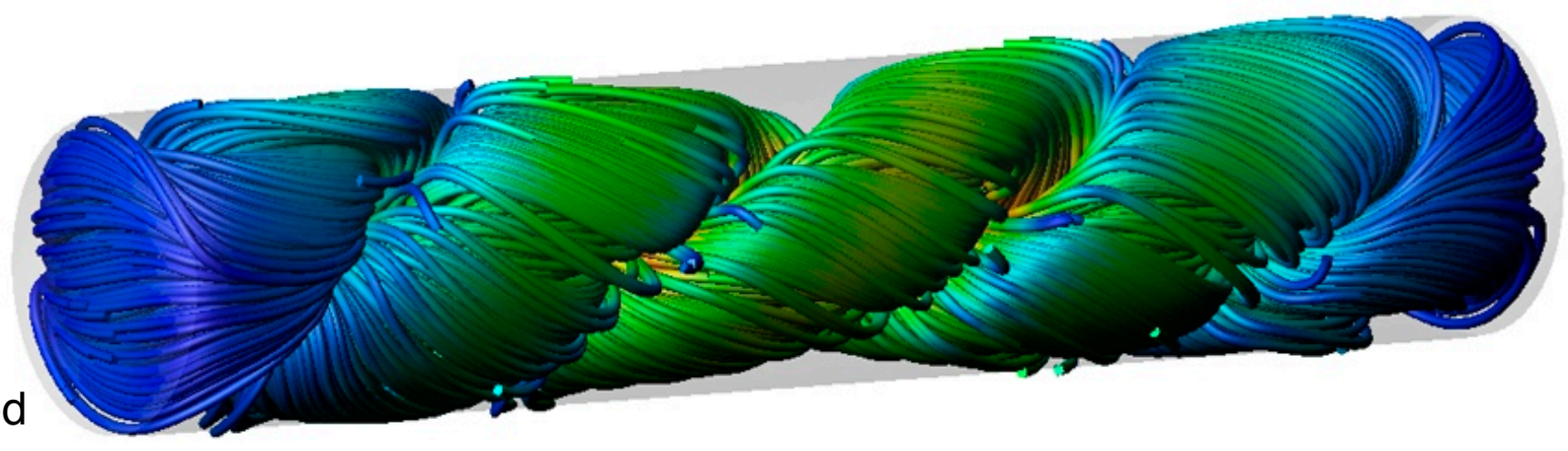


Figure. 6 Result from a simulation of the SSX plasma that shows the relaxed twisted Taylor state

In the twisted Taylor state shown above (Fig. 6), it can be seen that the twists of the magnetic field lines form "lobes" of plasma. These lobes are approximately 40 *cm* in length which corresponds to the spatial correlation period in the axial probe auto-correlation figure (Fig. 5). The axial auto-correlation plot shows the evolution of the plasma from a period of turbulent activity (25-40 μs) to a relaxed state (40-80 μs).

Radial probe measurements

The spatial auto-correlation measurement for the radial probes shows periodic bursts of high correlation across the entire probe length (~7 *cm*) in one direction (Fig. 7).

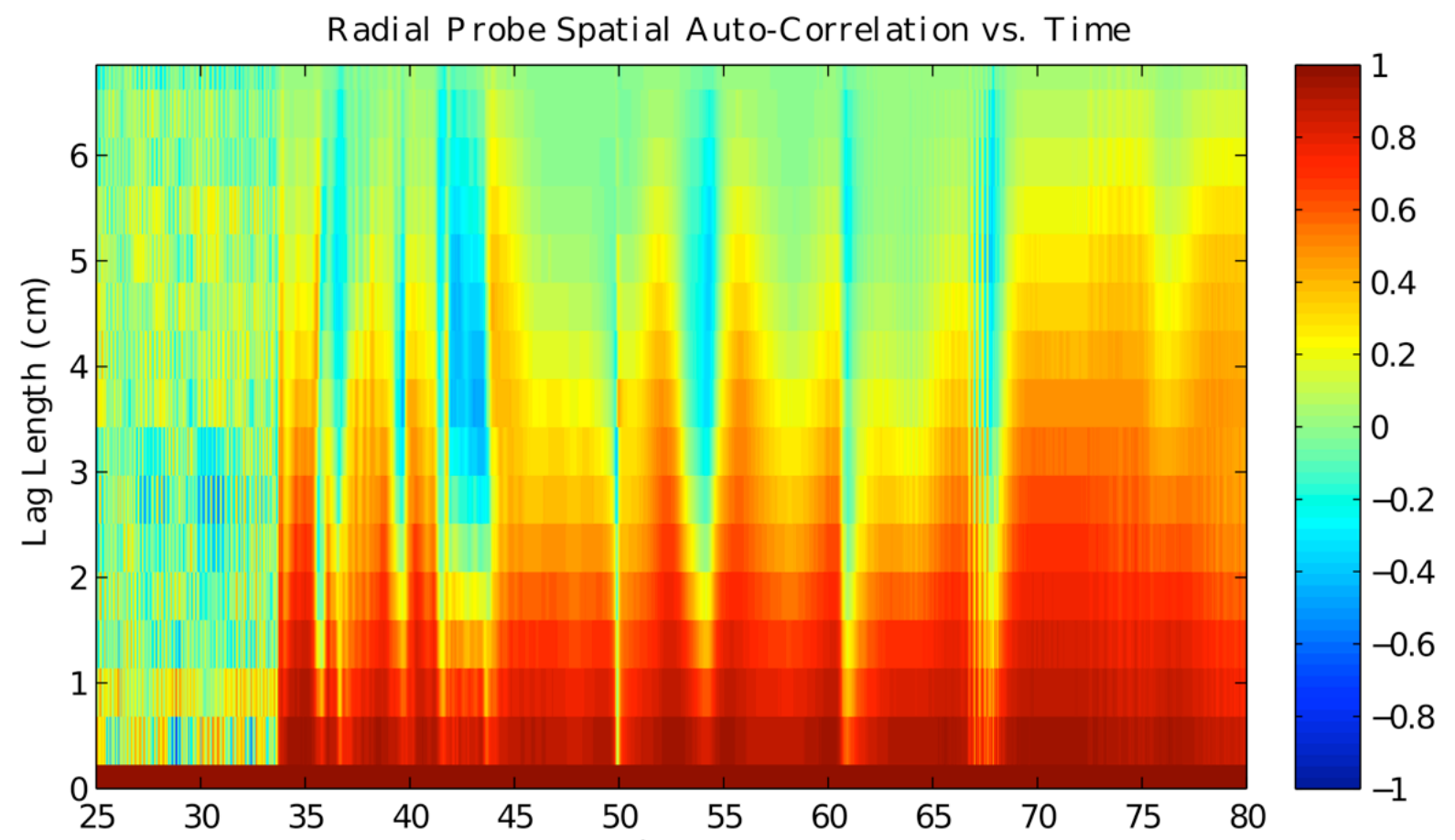
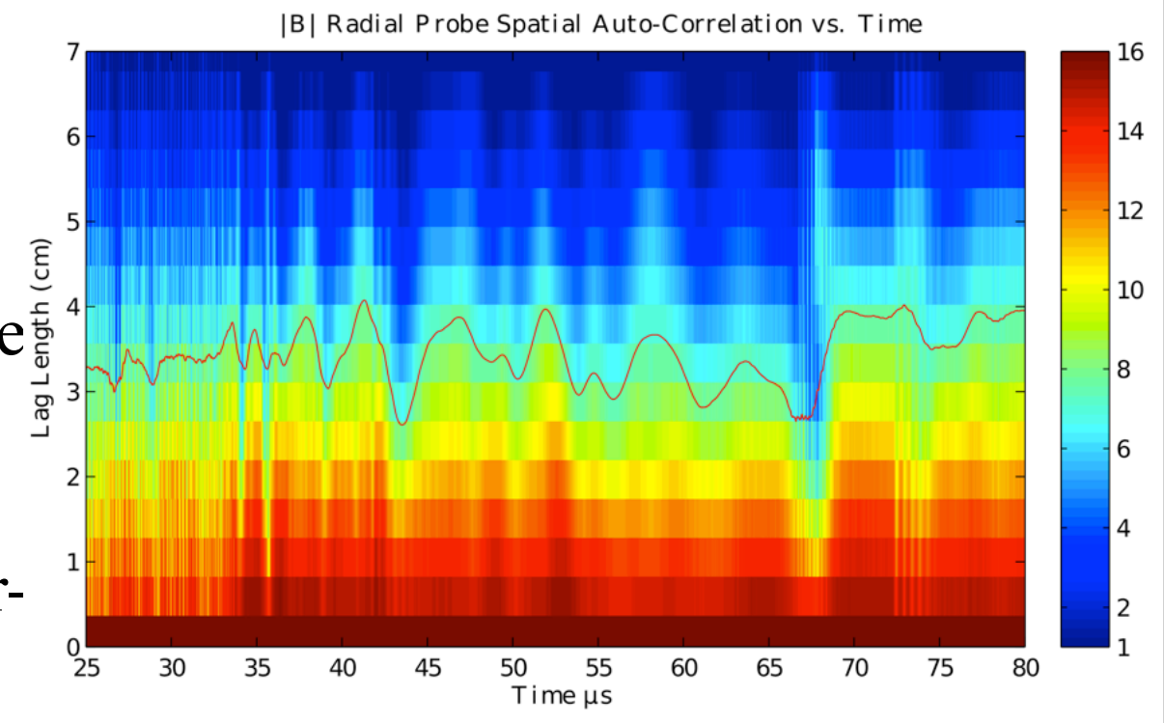


Figure. 7,8 Radial probe auto-correlation plot in the θ direction (left) and in the magnitude of **B**—along with a plot of the characteristic length vs. time—(bottom) showing the periodic bursts of high correlation in one direction and a sustained correlation in the magnitude of **B**



The length of the highly correlated bursts in a single direction suggests that there are turbulent structures ~4 *cm* in size rotating around the probe. A plot comparing the characteristic length of different probe orientations (Fig. 9) shows that at any time step at least one direction of the probe will be highly correlated. Additionally, a plot comparing the two probes (Fig. 10) shows a phase shift in the periods of high correlation between the two probes.

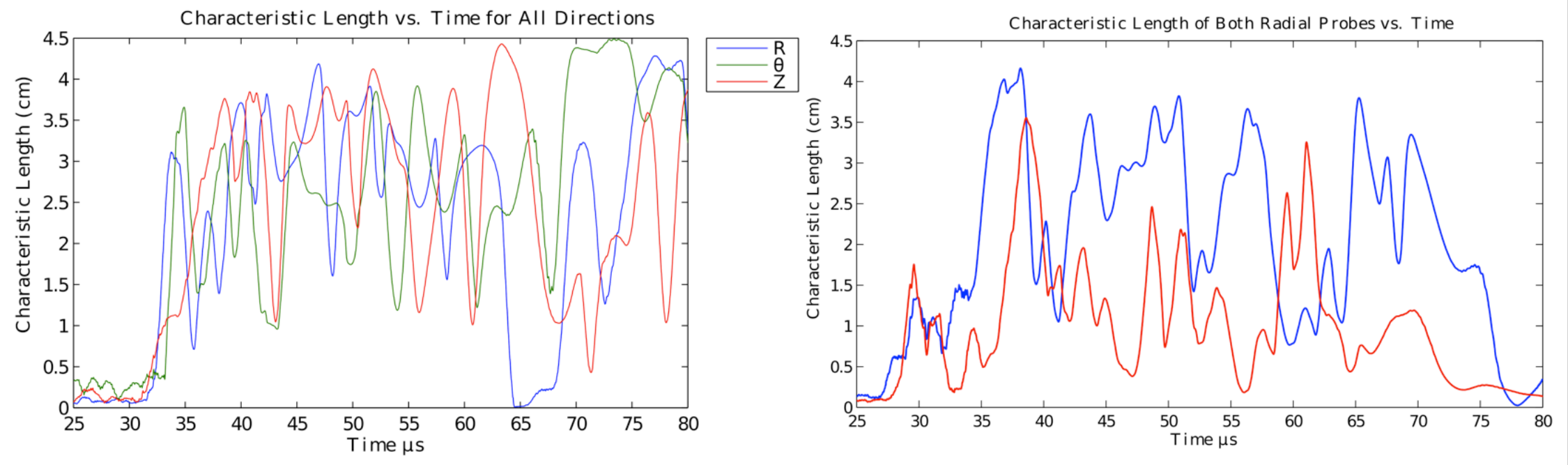


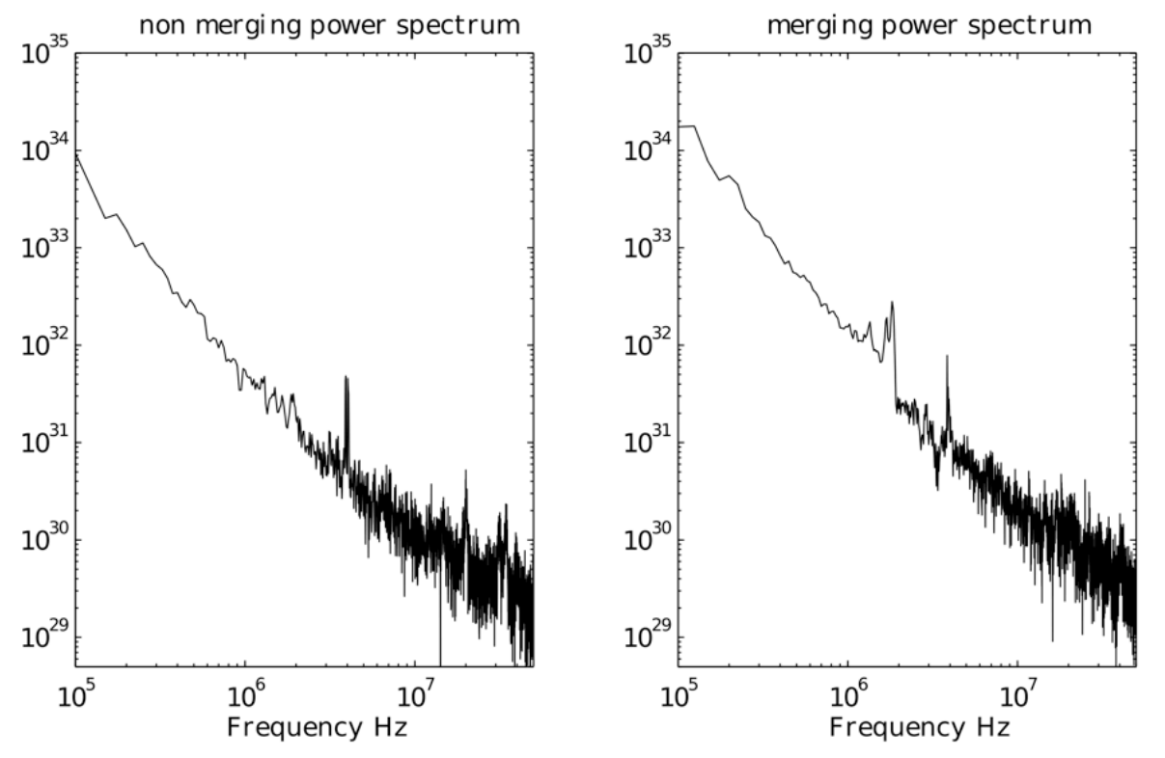
Figure. 9,10
Comparison of the characteristic length (Eq. 3) in time from both radial probes showing the shift from being similar early in time to being nearly opposite later (right) and a comparison of the three directions from a single probe (left)

Shifting of the relative phase later in time might point to the twisted Taylor state forming with one probe in a lobe and the other in a gap between lobes. The periodicity of the auto-correlation could indicate an azimuthal rotation of the relaxed spheromak.

Spectral analysis

Studies of fluctuations in the SSX plasma were made using the radial magnetic probes for magnetic field variations and a He-Ne interferometer for density fluctuations. The SSX device can be set up in a merging configuration to collide two spheromaks at the midplane or to fire a single spheromak down the flux conserver wind tunnel.

Figure. 11
Merging and non-merging power spectrum comparison from the He-Ne interferometer showing a feature around 2MHz during merging



Comparisons of merging and non-merging spectra from both the magnetic probes and the interferometer showed fluctuations around 2 *MHz* in the merging configuration that were absent in the non-merging runs.

Spectral analysis (cont.)

The ion cyclotron frequency of the SSX plasma is estimated to be approximately 2 *MHz*. This result suggests that these fluctuations could be representative of some sort of ion dynamics occurring during reconnection.

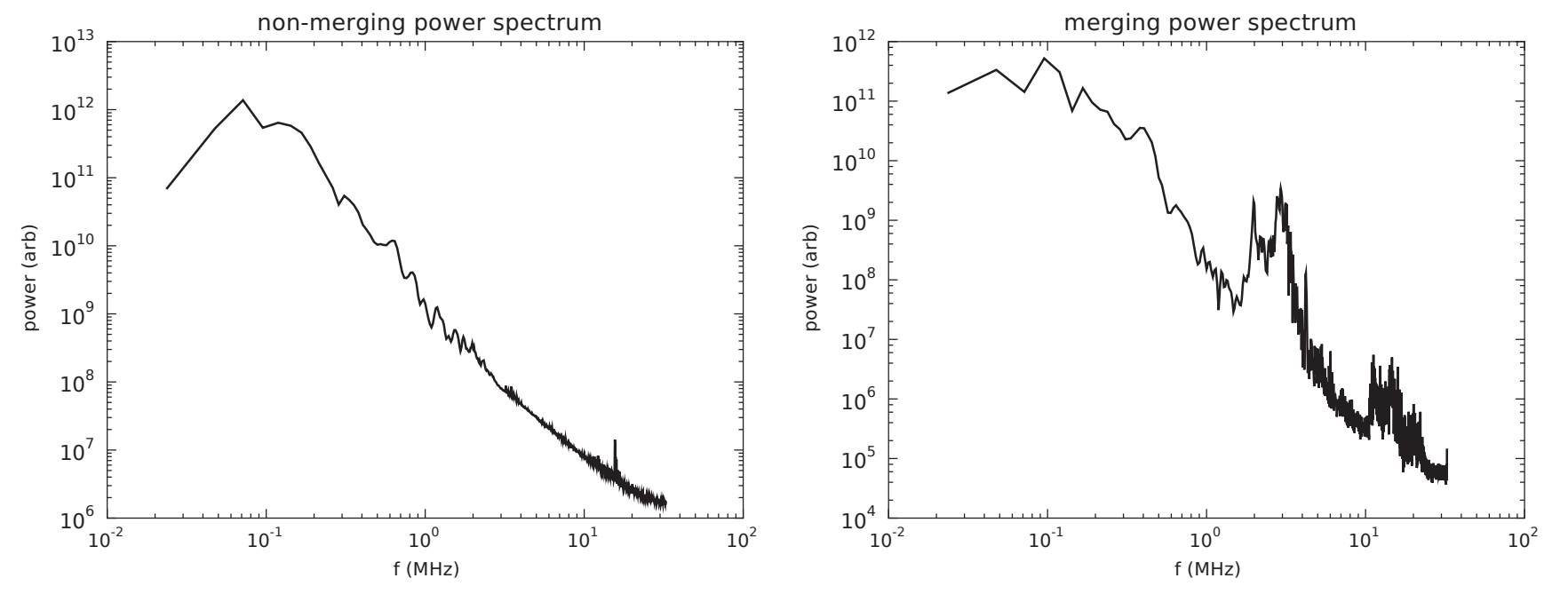


Figure. 12 Power spectra from the radial magnetic probe showing the same 2MHz feature present only in the merging case

Additionally, the interferometer was located off-midplane and still recorded a similar fluctuation (in density). This might be indicative of a plasma wave traveling outward from a reconnection event at the midplane.

Summary

New statistical techniques motivated from studies of the solar wind [1,2] were applied to signals from the magnetic diagnostics in SSX. The auto-correlation of signals from both the radial and axial probes showed evidence of moving structures in the SSX plasma that suggest turbulent motion. In order to continue the study of turbulence in the SSX wind tunnel, further refinement of statistical techniques is called for. Additionally, fluctuations in both the density and magnetic field during magnetic reconnection should be explored further in hopes of finding their cause.

Literature cited

[1] W.H. Matthaeus, S. Dasso, J.M. Weygand, L.J. Milano, C.W. Smith, and M.G. Kivelson. *Spatial correlation of solar-wind turbulence from two-point measurements*. Phys. Rev. Lett. **95**, 231101 (2005).

[2] W.H. Matthaeus, S. Dasso, J.M. Weygand, M.G. Kivelson, and K.T. Osman. *Eulerian decorrelation of fluctuations in the interplanetary magnetic field*. APJ. 721 L10 (2010).

Acknowledgments

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For further information

Please contact `kflanal@swarthmore.edu`. More information on SSX can be obtained at **`http://plasma.swarthmore.edu/SSX/index.html`**