

MHD turbulence: observation and experiment

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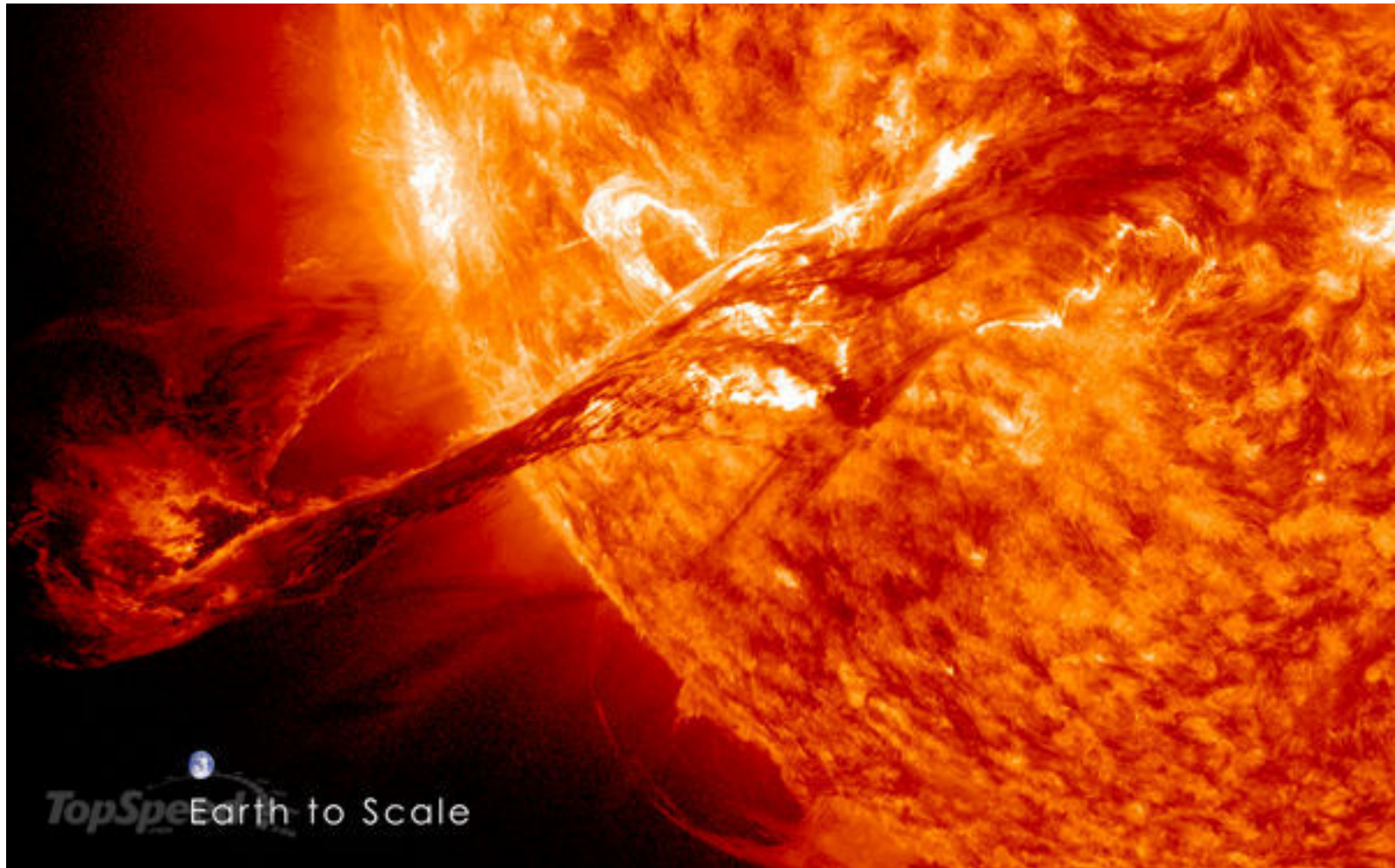
with contributions from

D. Schaffner, V. Lukin, P. Weck '14, A. Wan '14, R. Wicks

APS-DPP tutorial
October 29, 2014

Research supported by US DOE and NSF

Solar plume (CME) and wind (plasma)



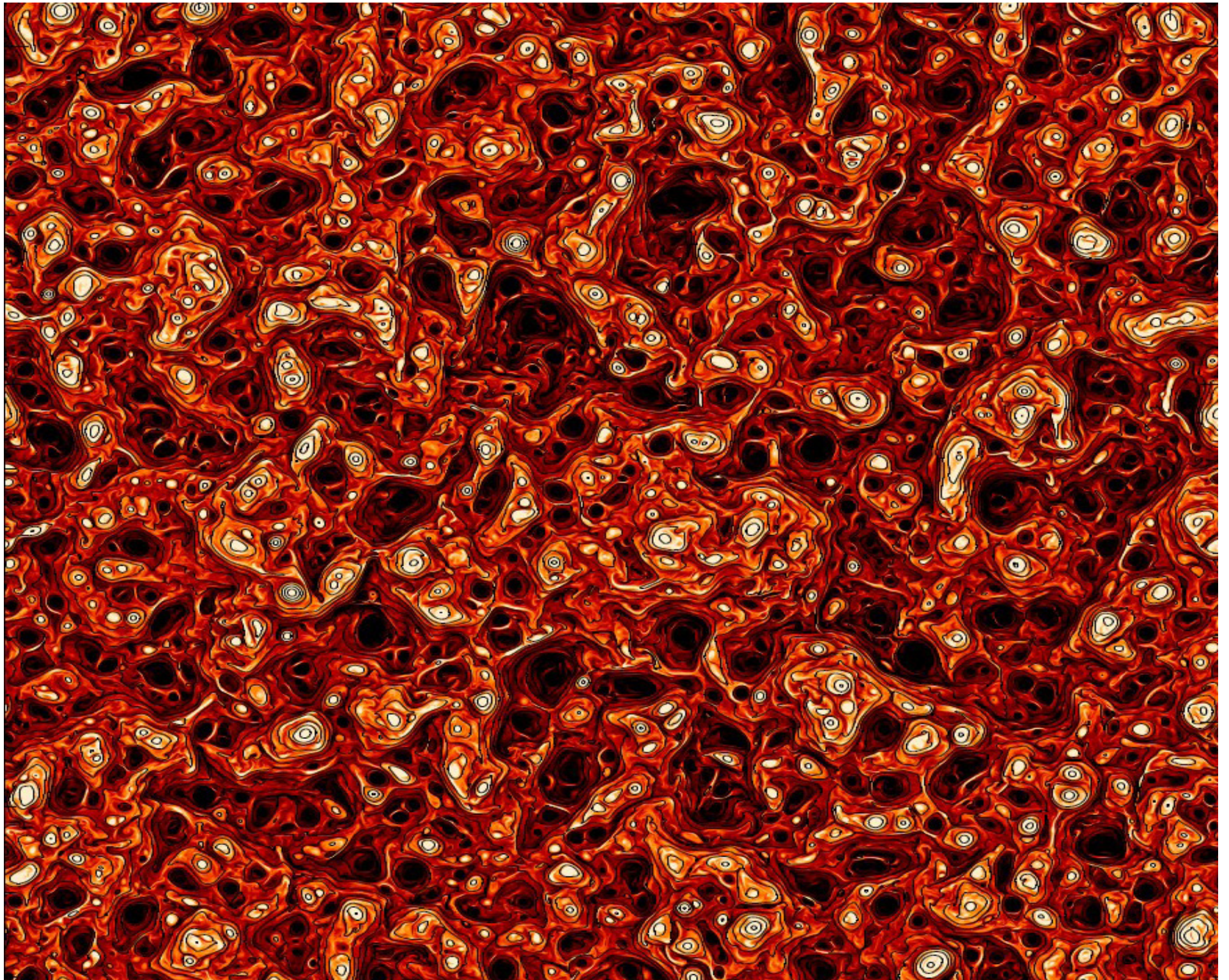
400 km/s plasma with entrained dynamical magnetic fields
(SDO, 8/12)

Outline

- Turbulence primer
- SSX plasma wind tunnel configuration
 - Four statistical measures for MHD turbulence

Turbulence Primer

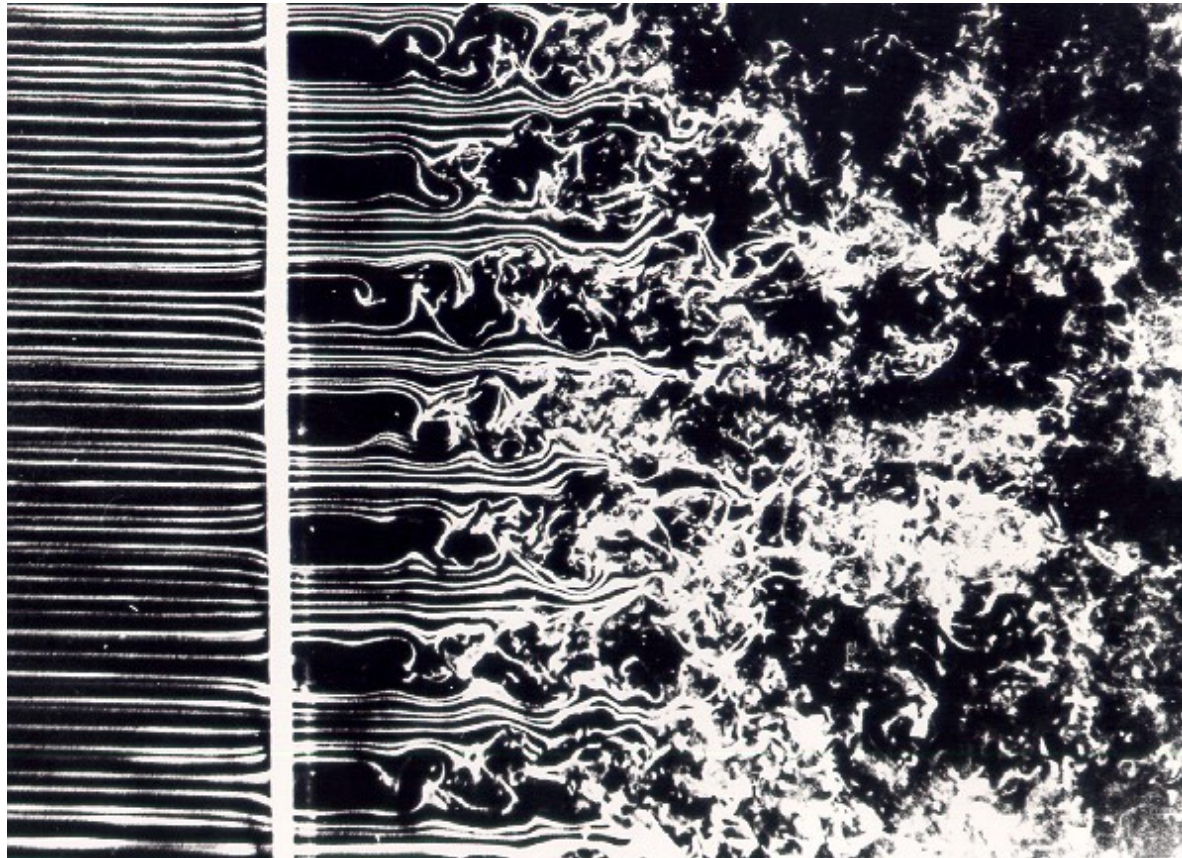
- Cascade from large to small scales
- Homogeneous, Isotropic, Stationary
- Universality in conventional fluids



Energy cascade

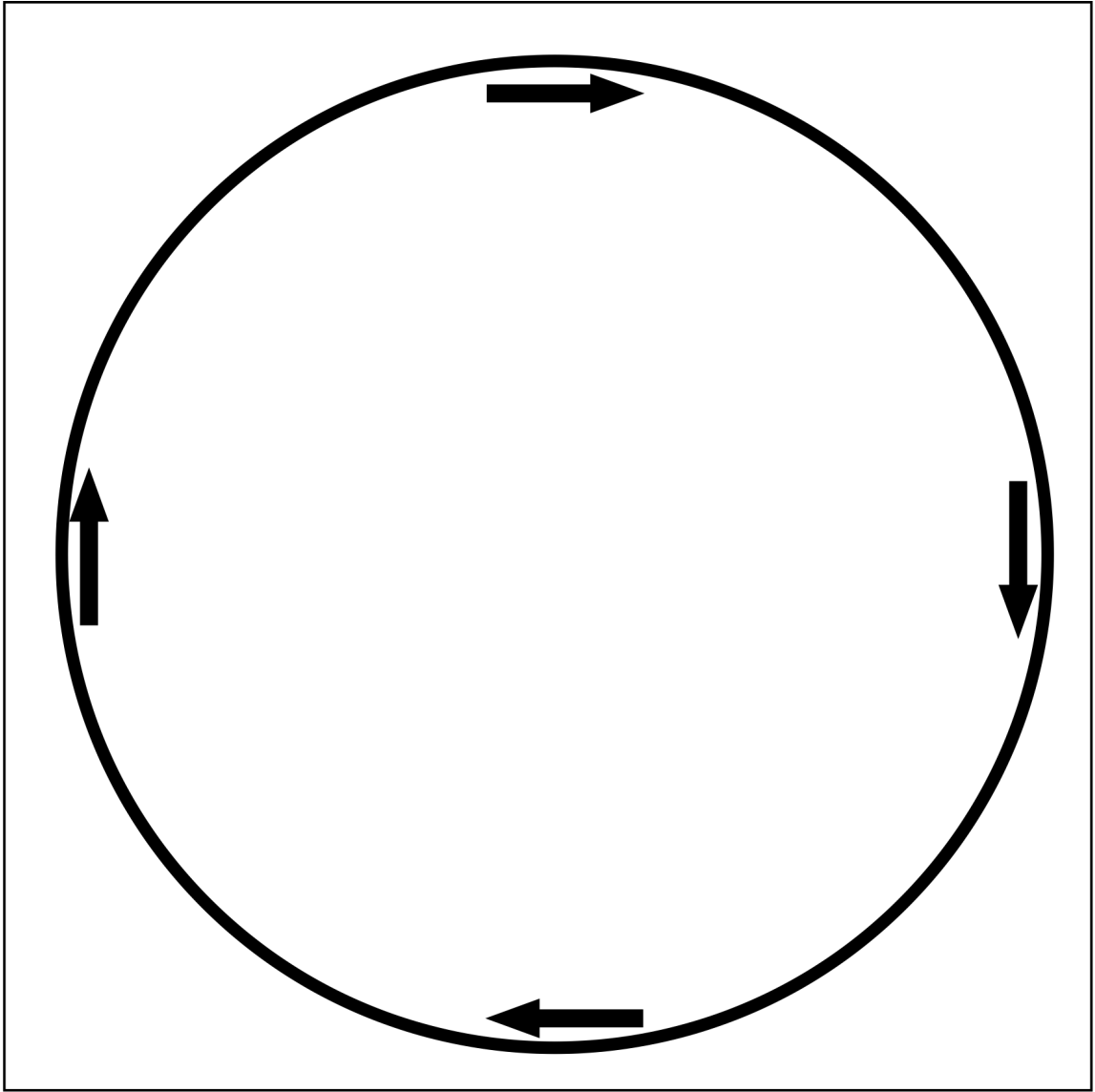


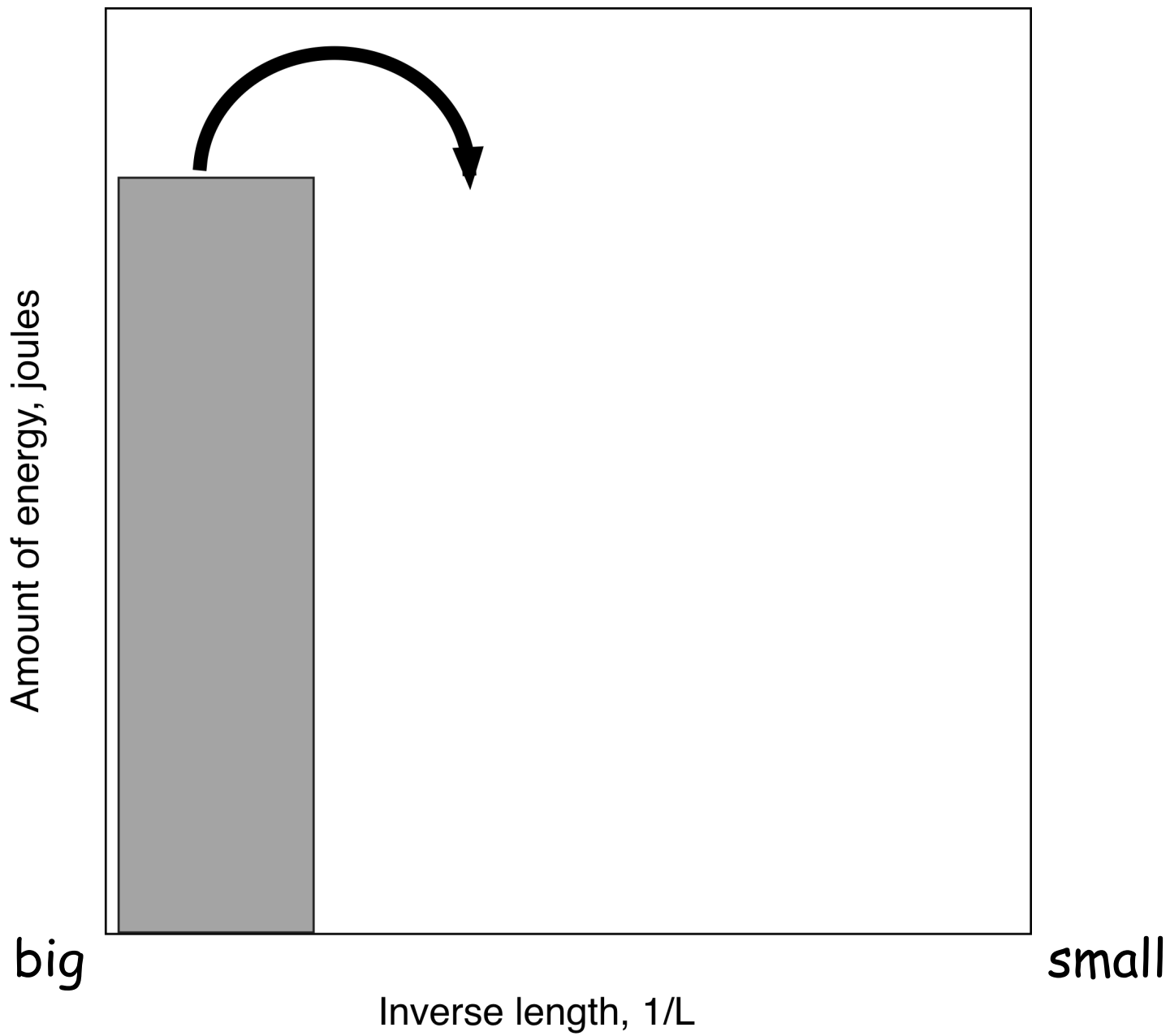
Grid turbulence

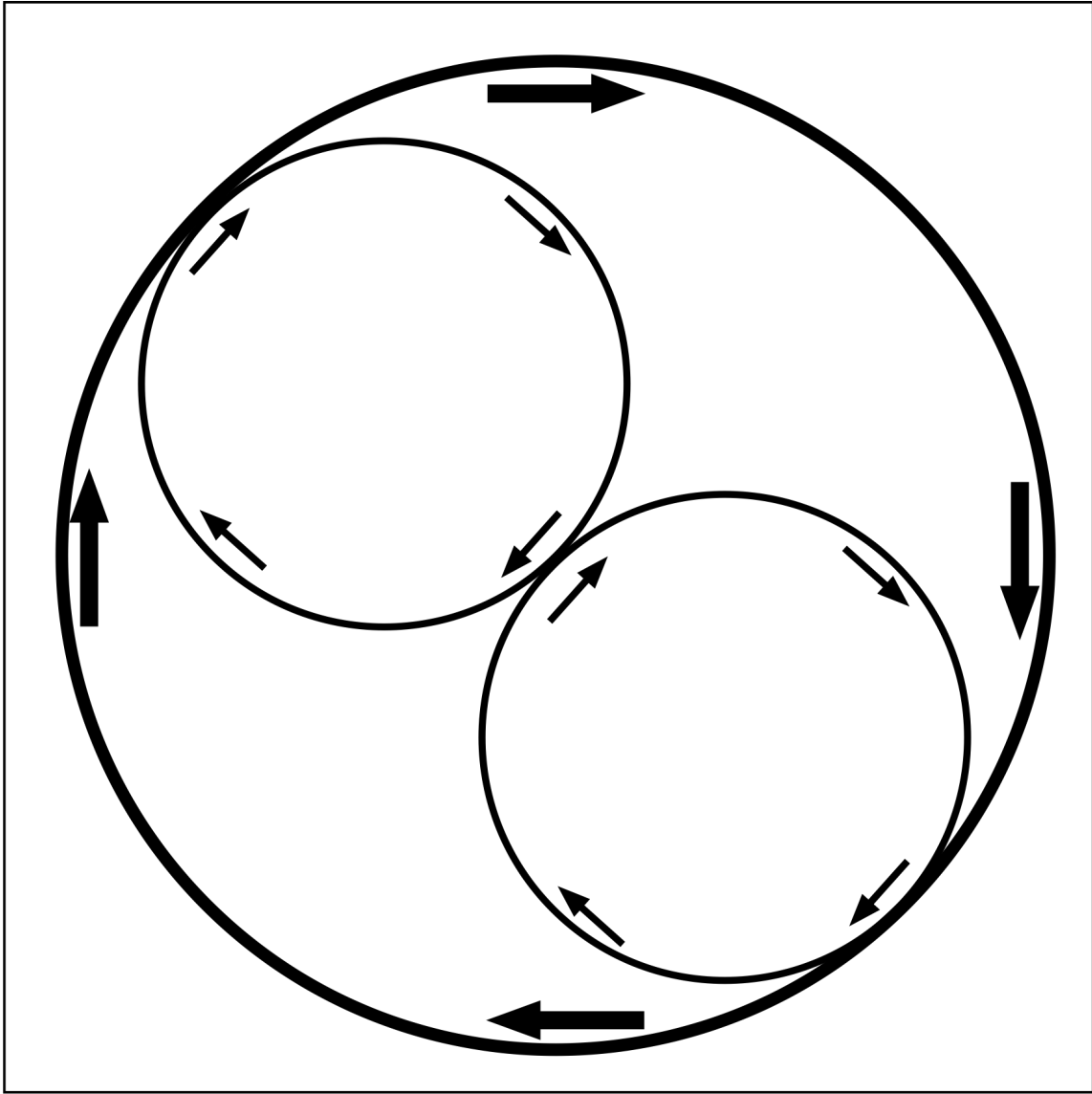


Fluid Dynamics Research Center

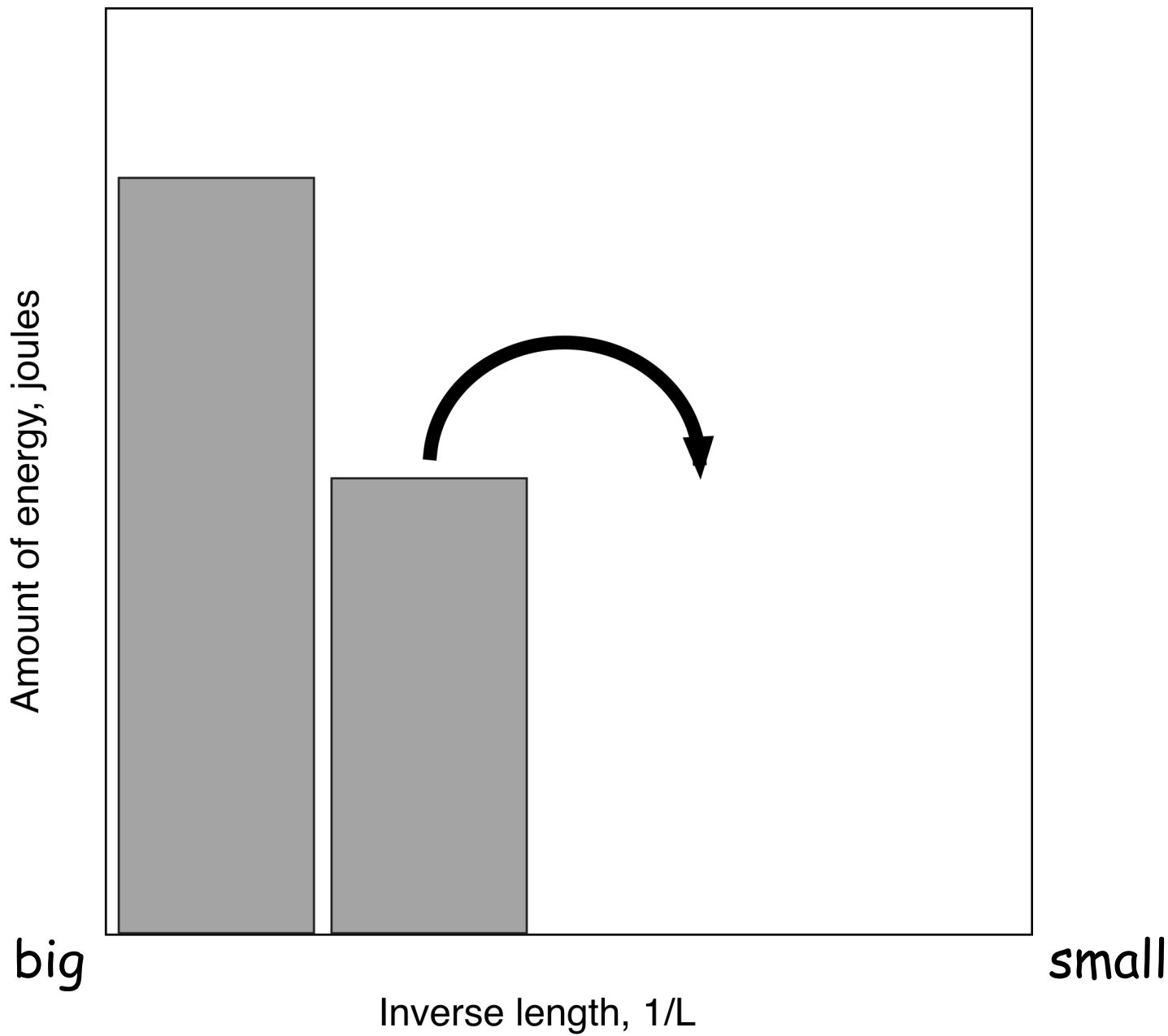
Note: Taylor hypothesis

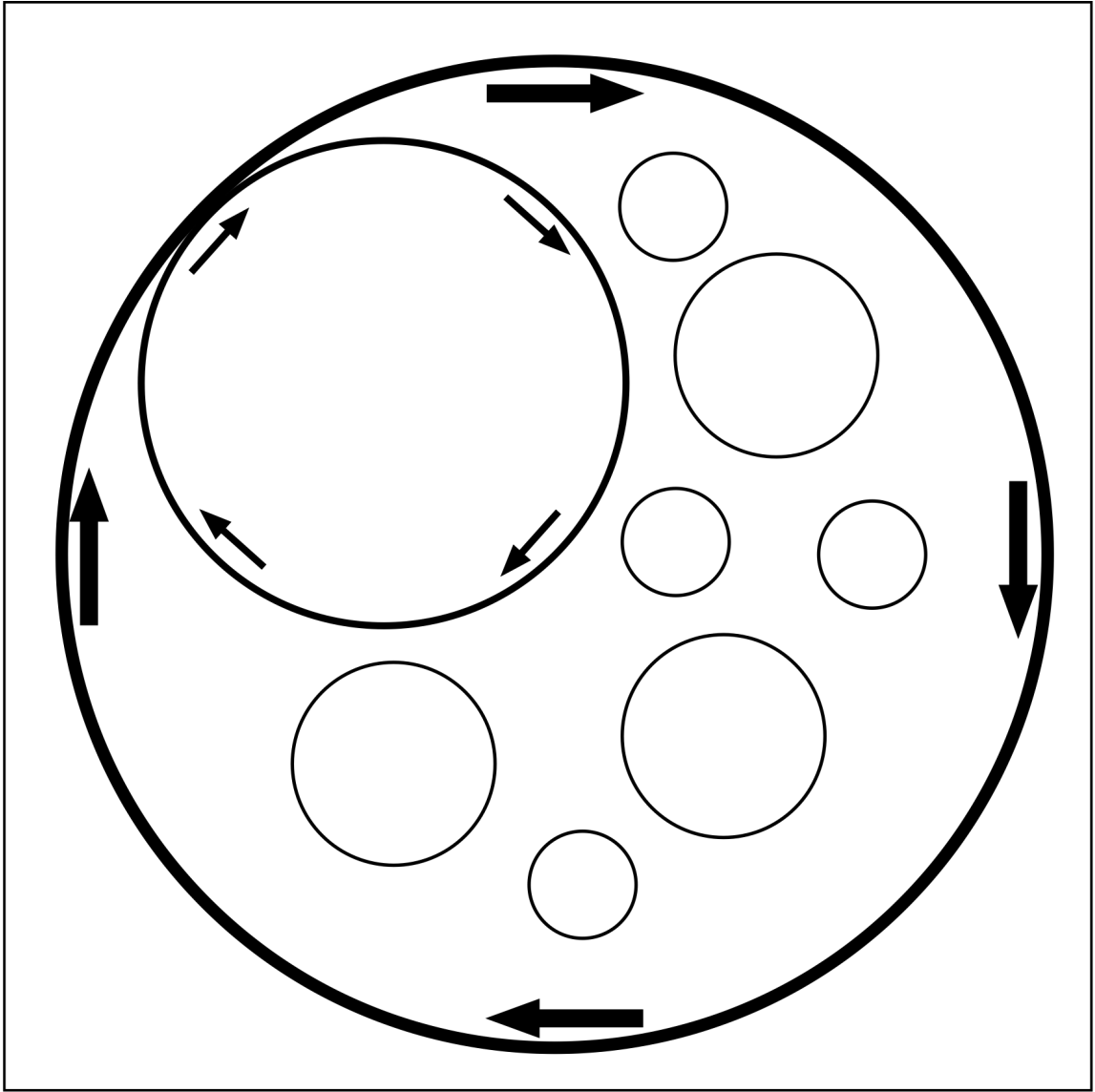


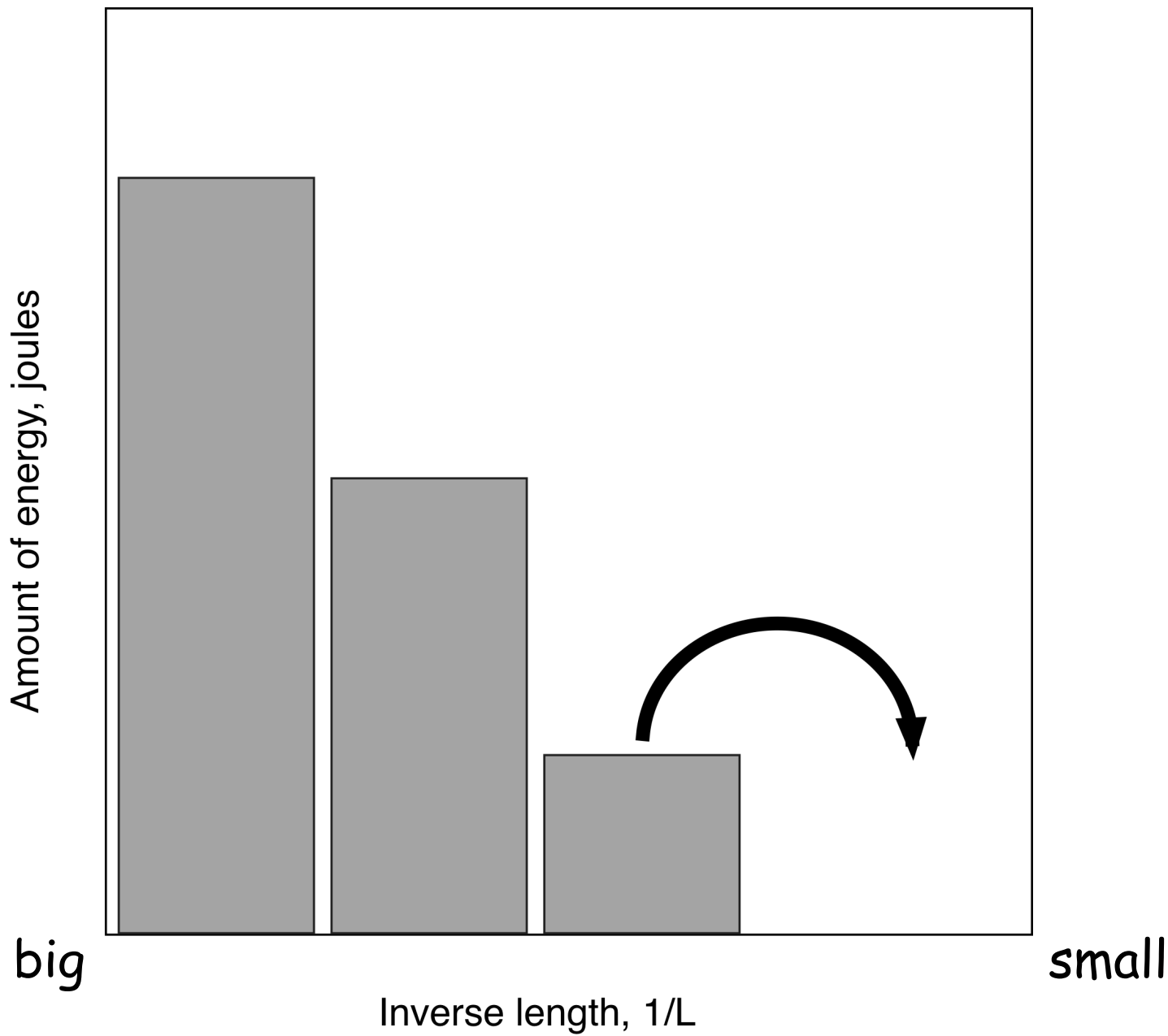




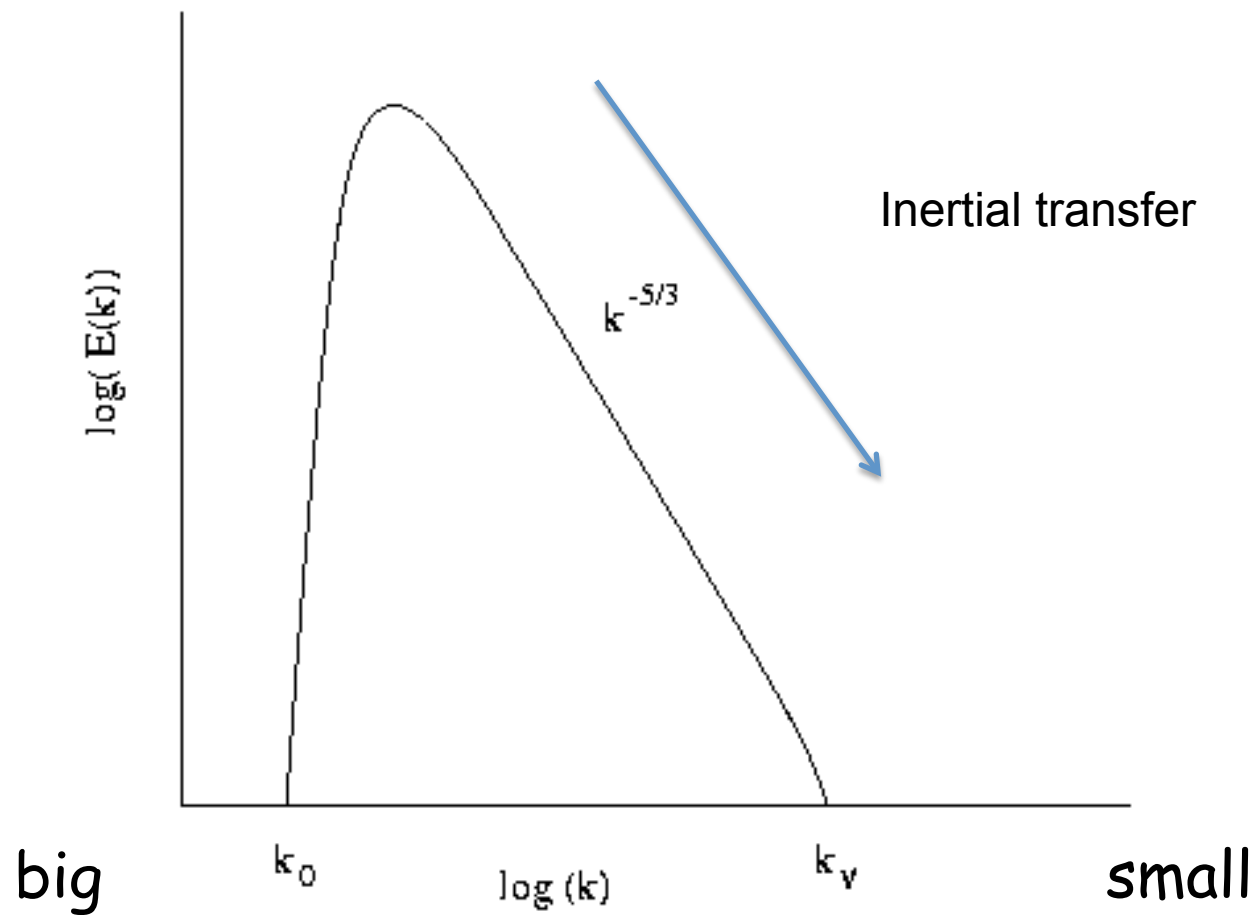
L



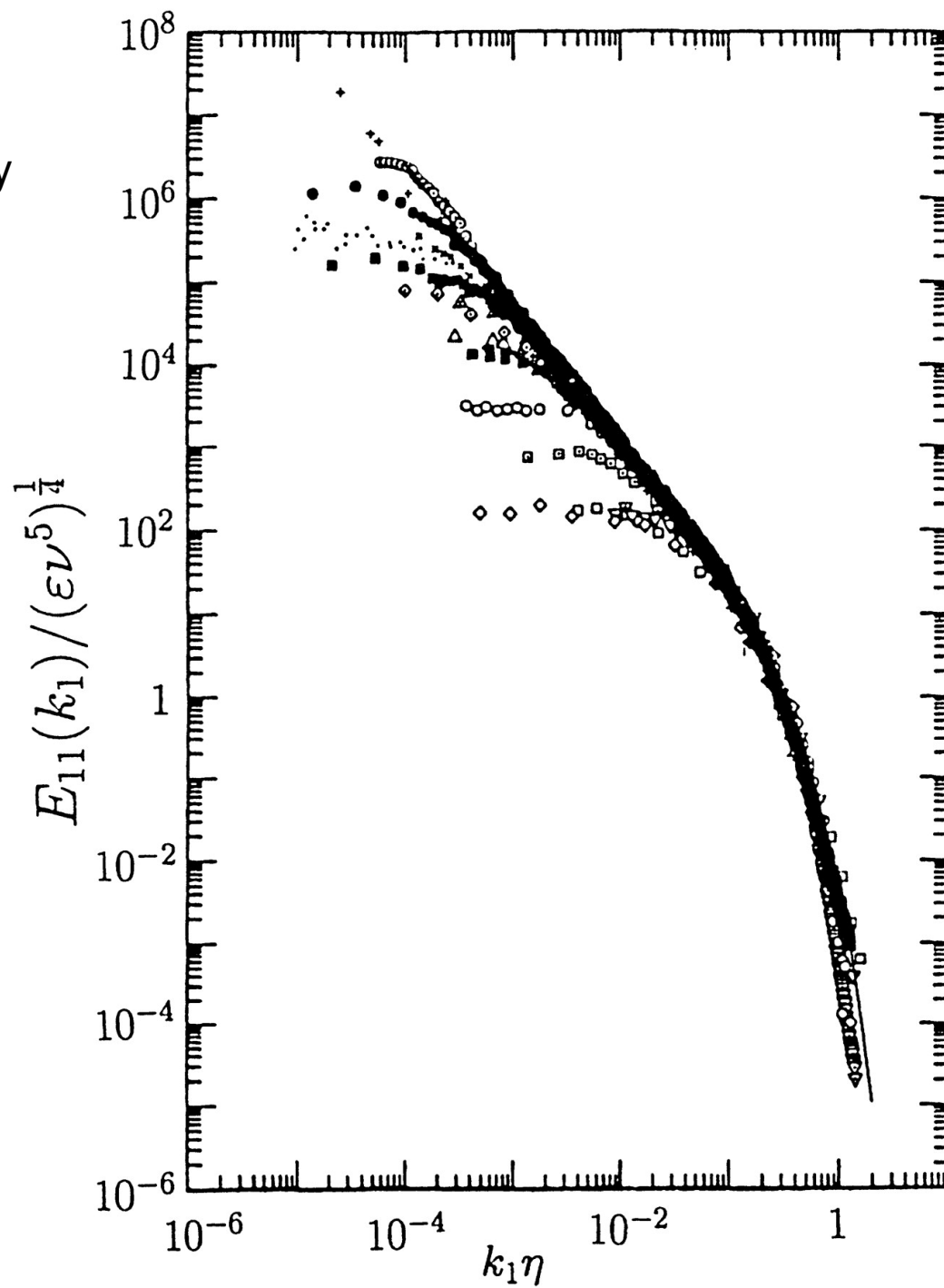


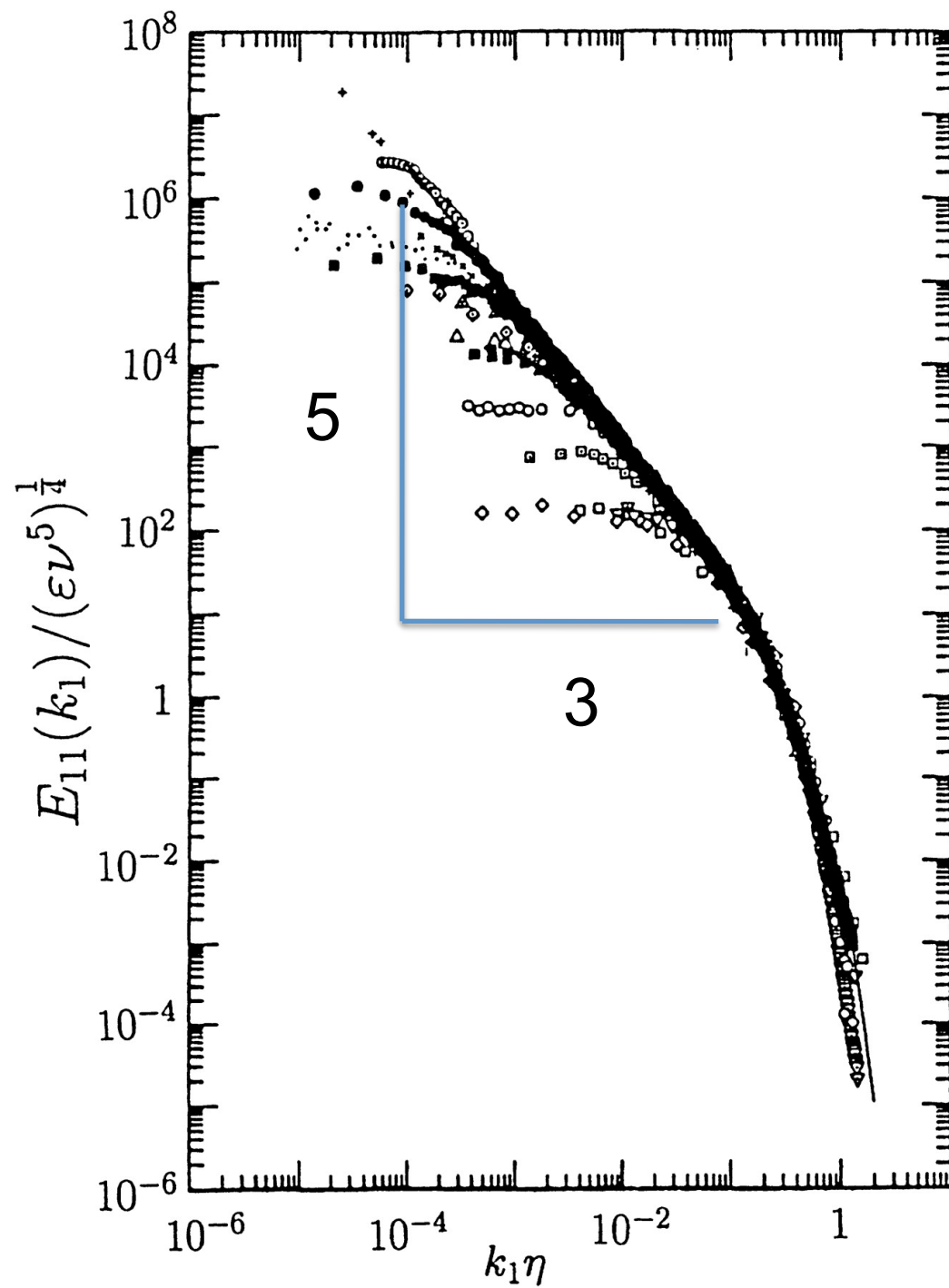


Kolmogorov turbulence spectrum (1941)



universality





Kolmogorov spectrum (K41)

$$E(k) = C\epsilon^\alpha k^{-\beta}$$

Depends only on transfer rate and k

$$E(k) \propto v^2 / k$$

Energy per mass per wavenumber

$$\epsilon \propto kv^3$$

Energy per mass per time

$$v^2 k^{-1} \propto k^\alpha v^{3\alpha} k^{-\beta}$$

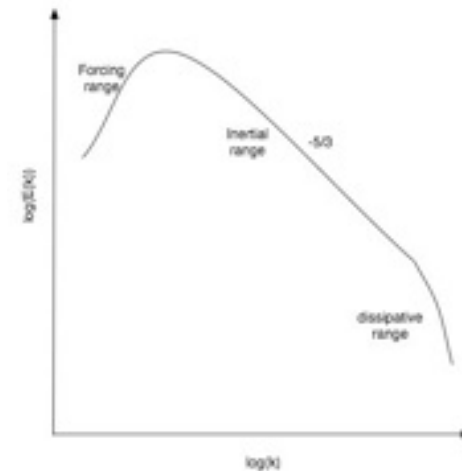
Dimensional analysis

$$2 = 3\alpha$$

$$-1 = \alpha - \beta$$

$$\alpha = 2/3$$

$$\beta = 5/3$$

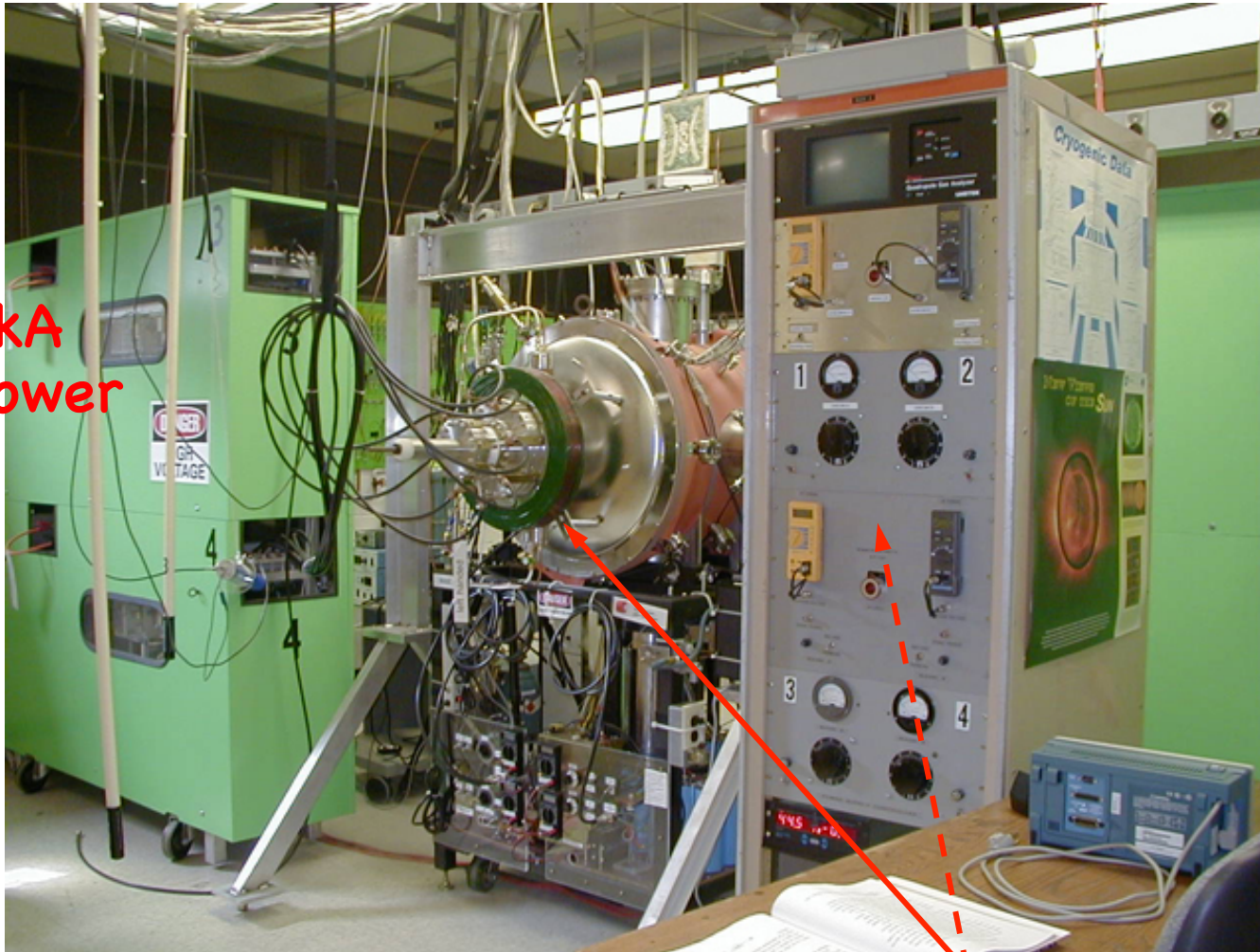


Turbulence Summary

- Turbulence in conventional fluids usually begins at large scales and “cascades” to small
 - The structure is usually formless, homogeneous, isotropic, stationary
- The spectrum of turbulence has a universal character ($k^{-5/3}$) in conventional fluids, also observed in magnetofluids

The SSX Laboratory

10kV/100kA
Pulsed power



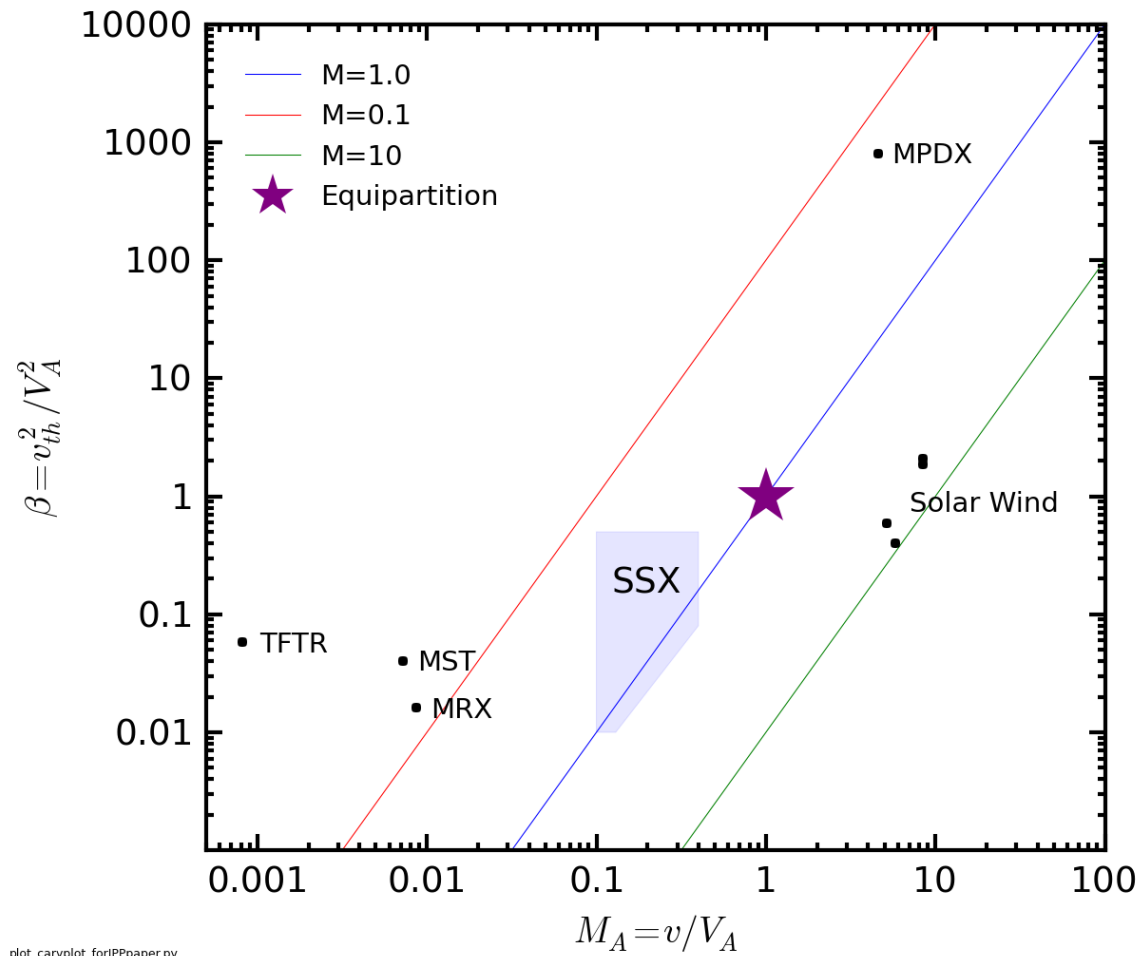
Cylindrical vacuum chamber
($D = 0.5$ m, $L = 1$ m)

High voltage plasma
guns on each end

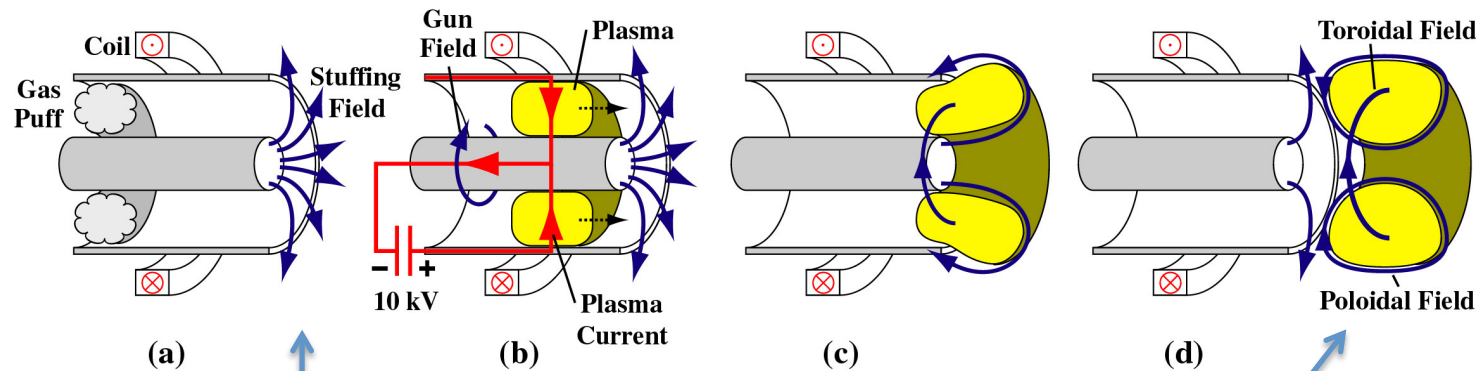
SSX parameters

Ion Density (protons)	$10^{14} - 10^{15} \text{ cm}^{-3}$
Temperature (T_e, T_i)	20 - 60 eV
Magnetic Field	> 0.1 Tesla
Ion gyroradius	< 0.5 cm
Alfvén speed	100 km/s
S (Lundquist number)	> 1000
Plasma β	0.1-1

Equipartition of flow, thermal, and magnetic energy



Spheromak formation

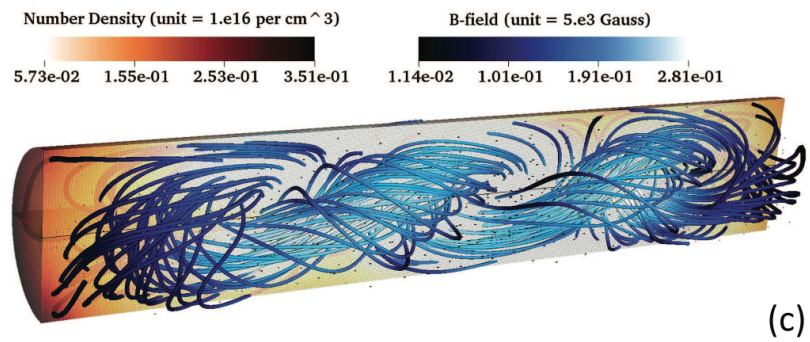
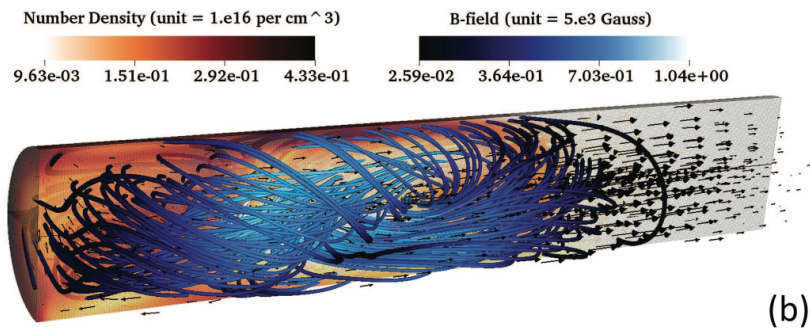
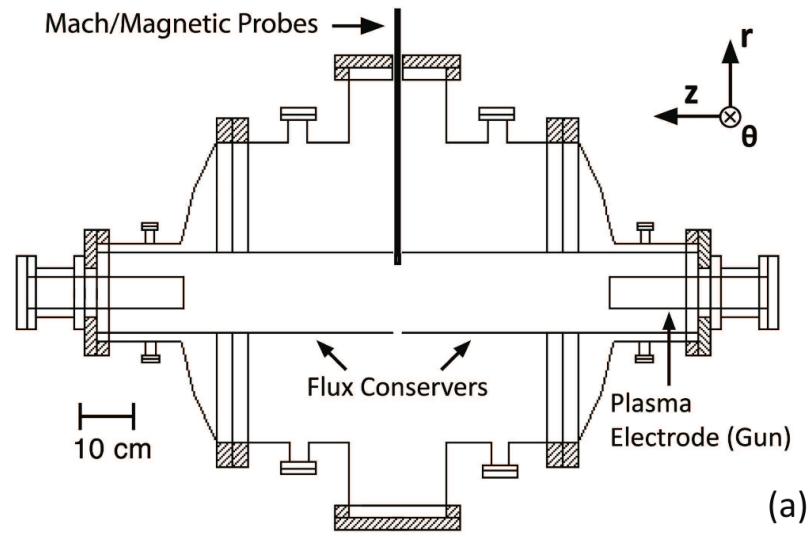


Spheromak

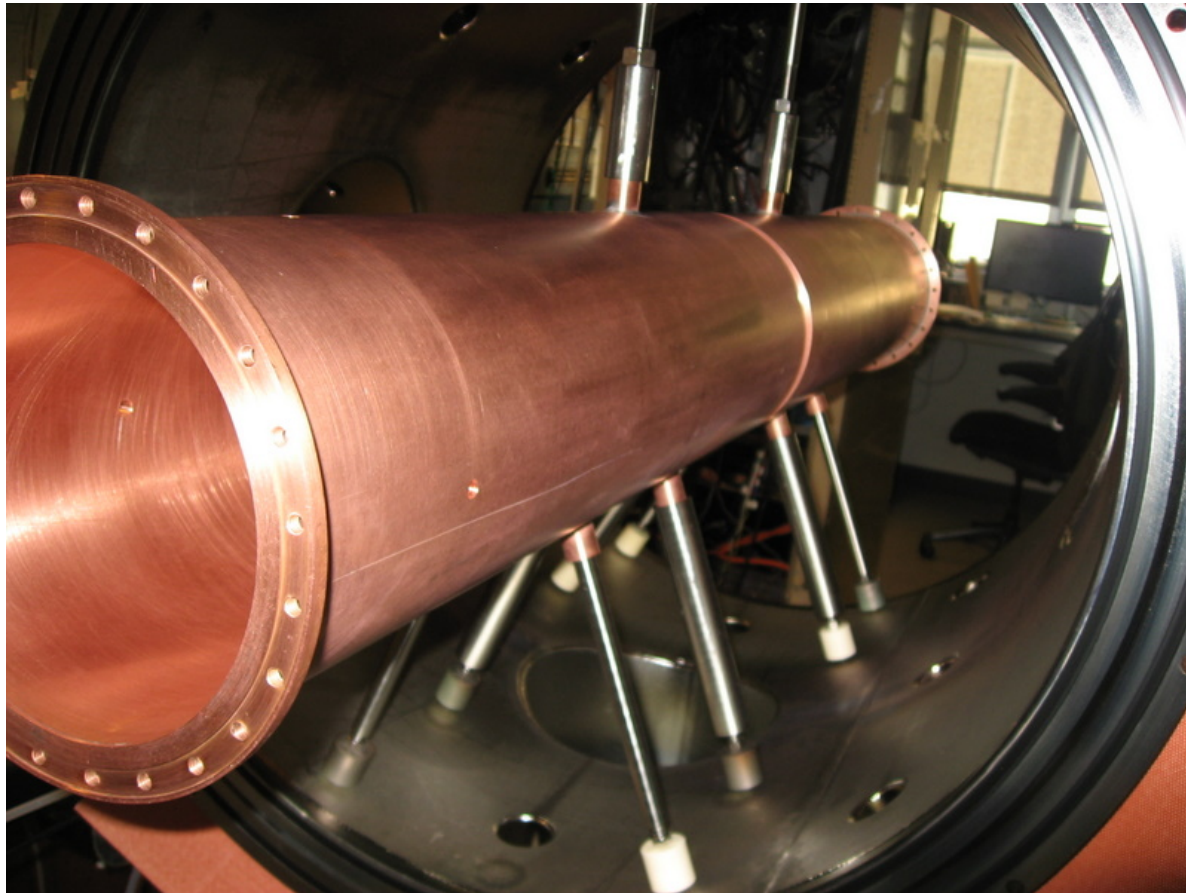
Stuffing flux acts like a nozzle

MHD wind tunnel

- 50 km/s flows, fully ionized and magnetized
- Kinetic, magnetic, thermal energies comparable
 - Single plume (>5 kJ)
- Characterization of MHD turbulence
 - Compare to solar wind

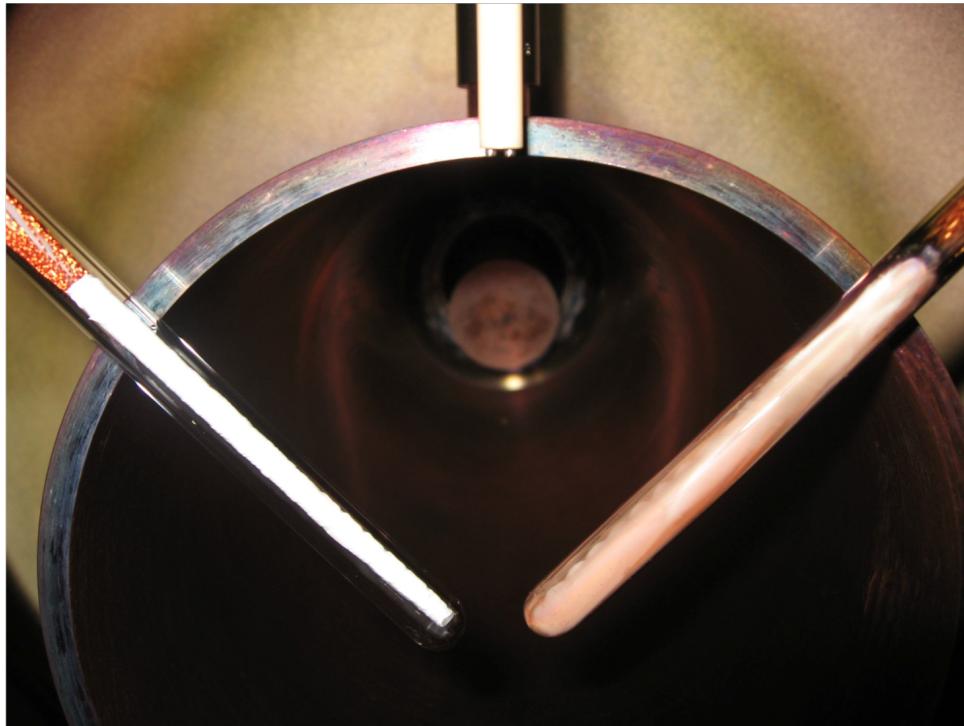


SSX MHD wind tunnel 50 km/s, magnetic and fluid turbulence



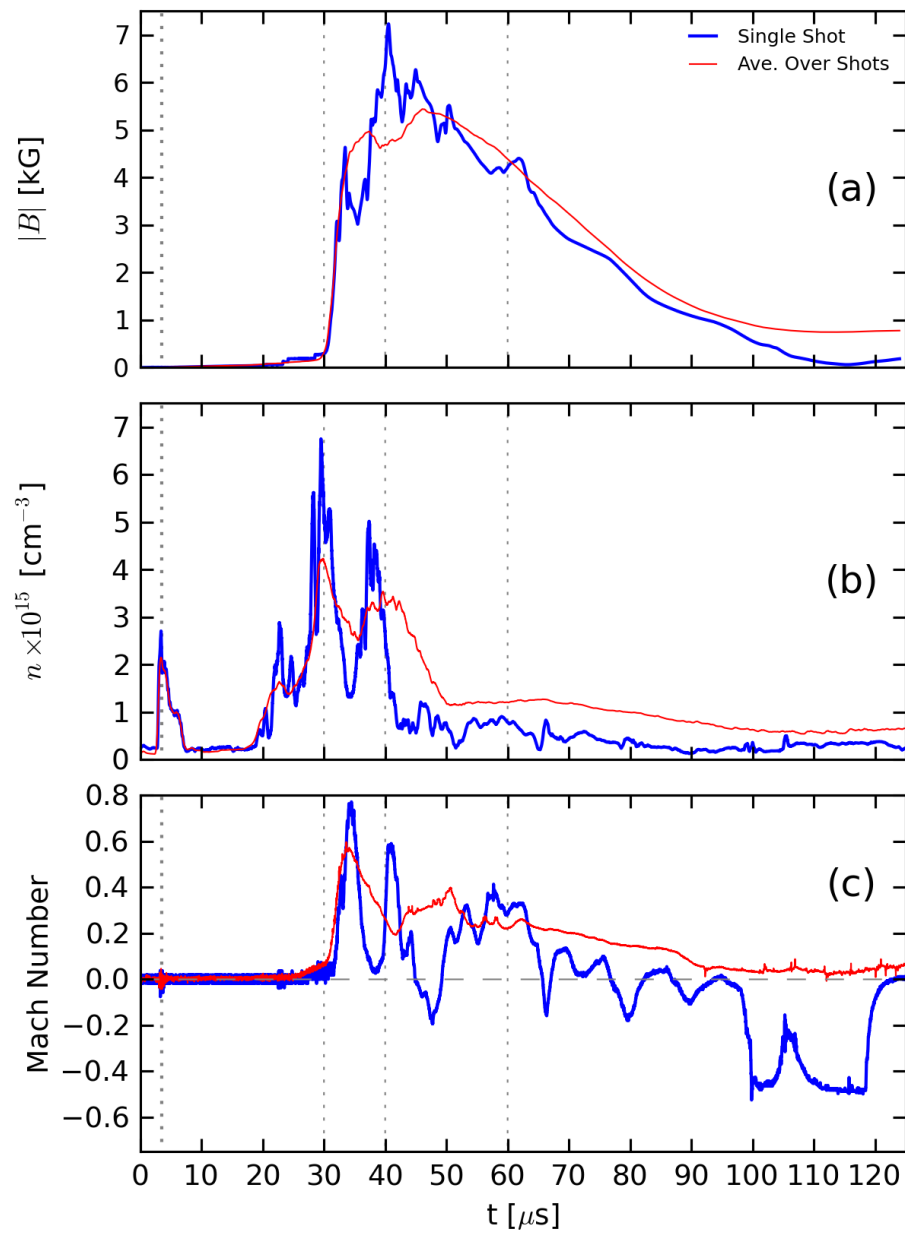
Diagnose with arrays of magnetic and velocity probes

Diagnostics at midplane (B and n_e)



Line-averaged density with He-Ne, temperature from IDS

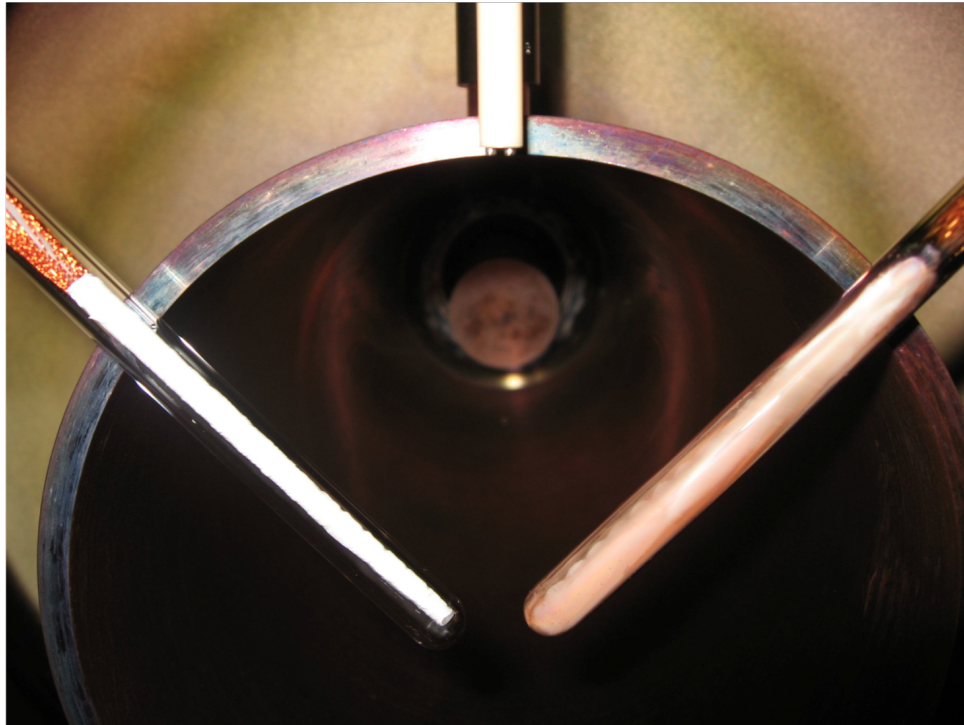
Formation/Selective Decay → ← Equilib. → ← Dissipation

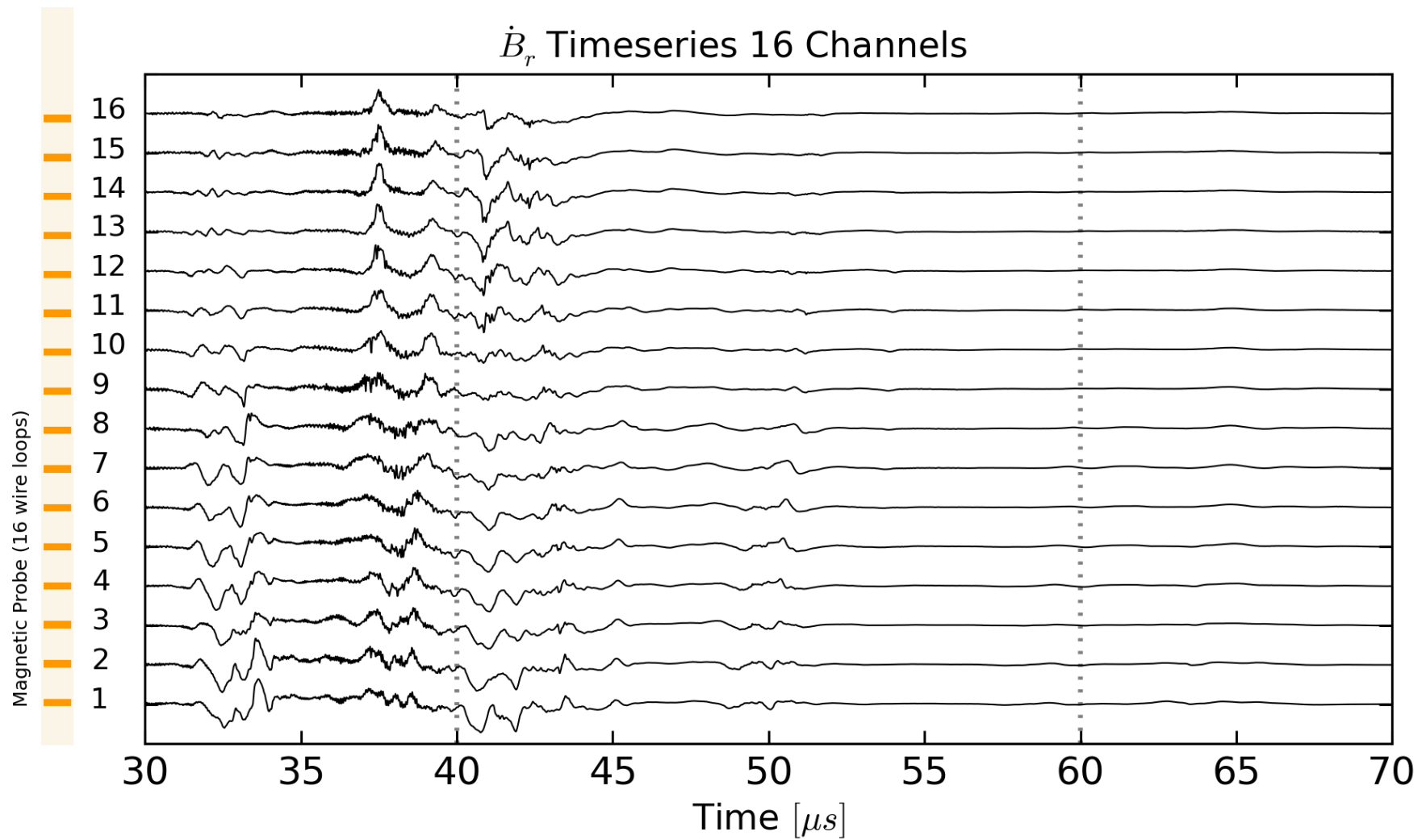


Four statistical metrics (SSX and solar wind)

- Frequency power spectrum $E_B(f)$
- PDF of temporal increments Δb
- Spatial correlation function $R(r)$
 - Permutation entropy C-H
- many other metrics are possible

Diagnostics at midplane (B and n_e)



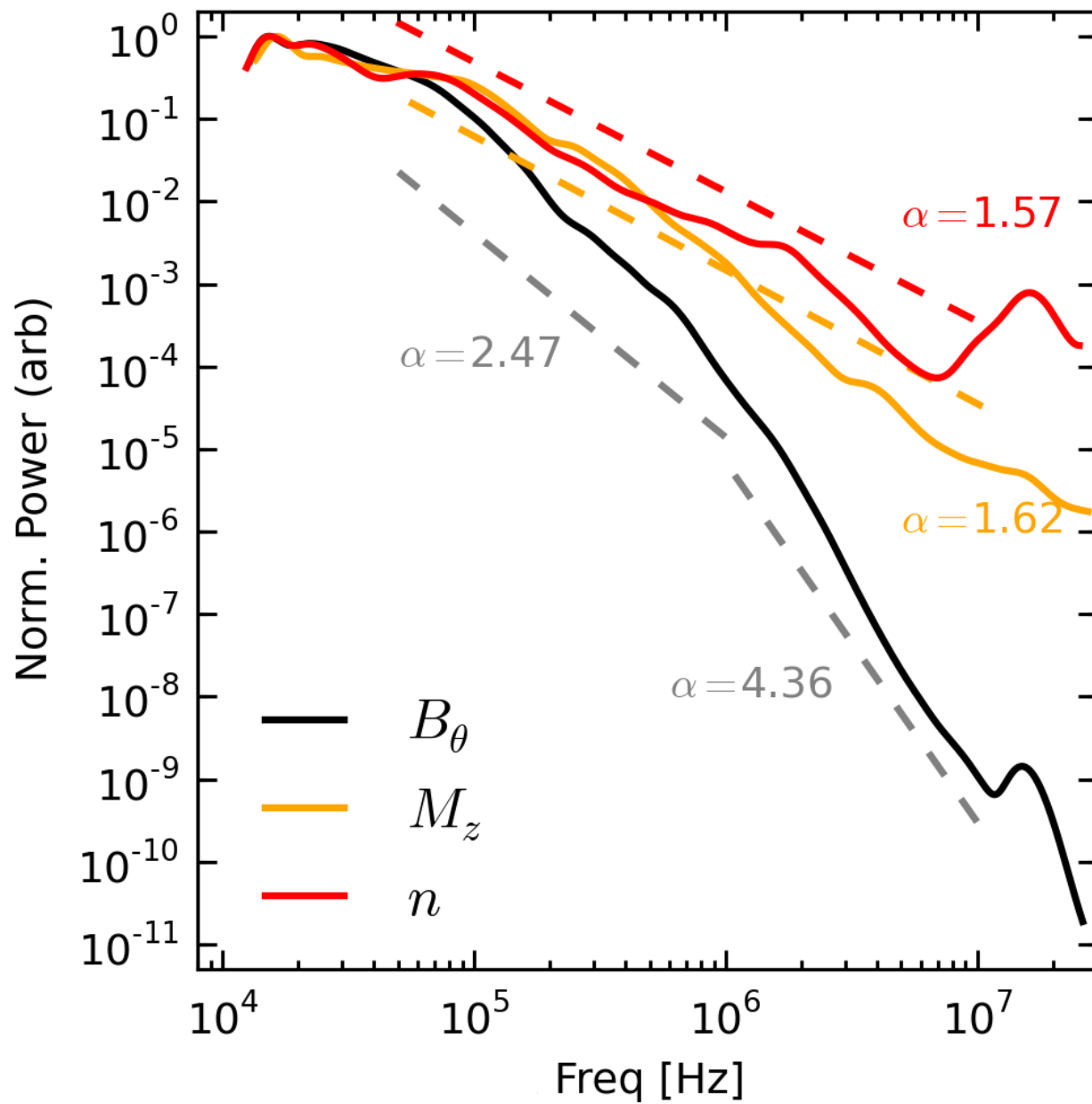


16 channels, 4 mm spacing, 65 MHz, 14 bit

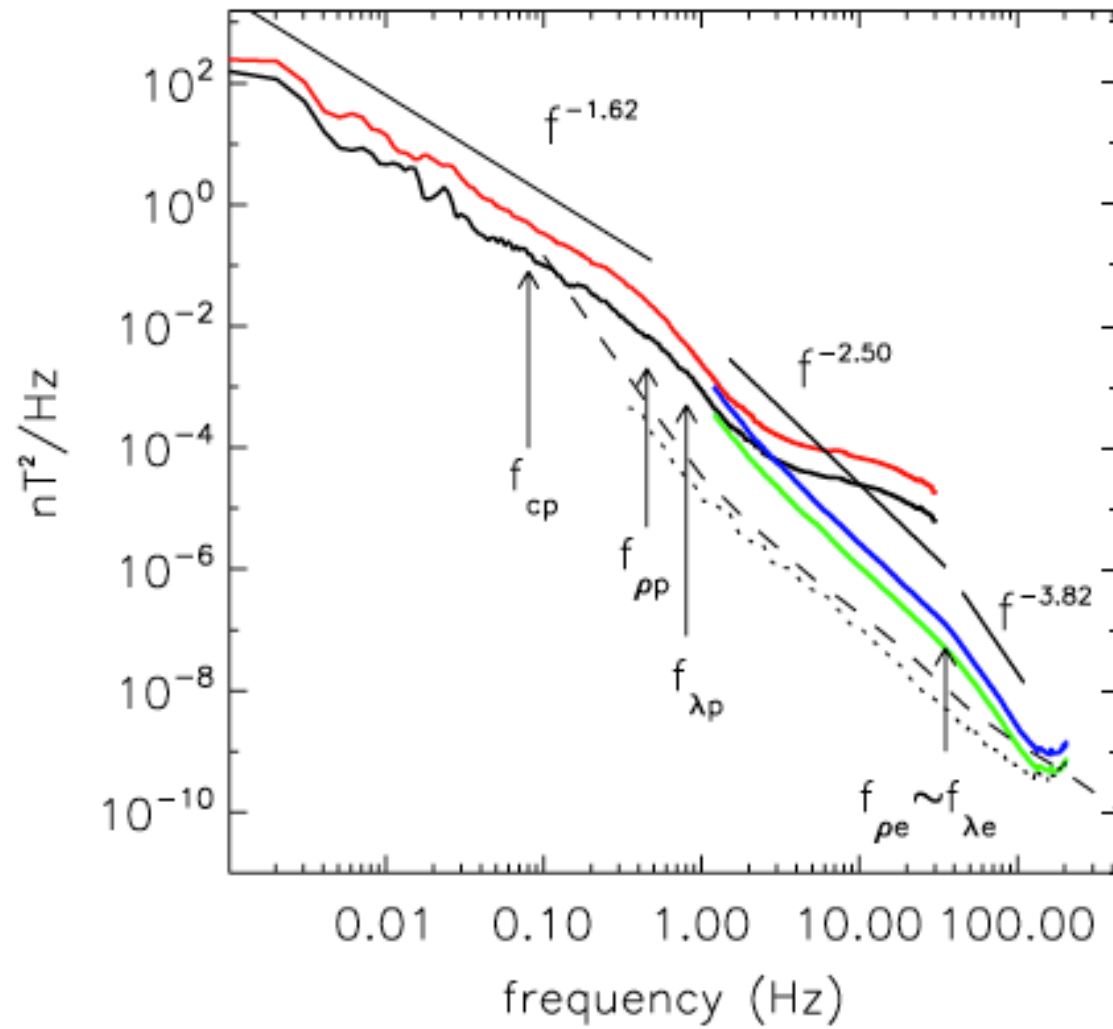
Frequency power spectrum $E_B(f)$

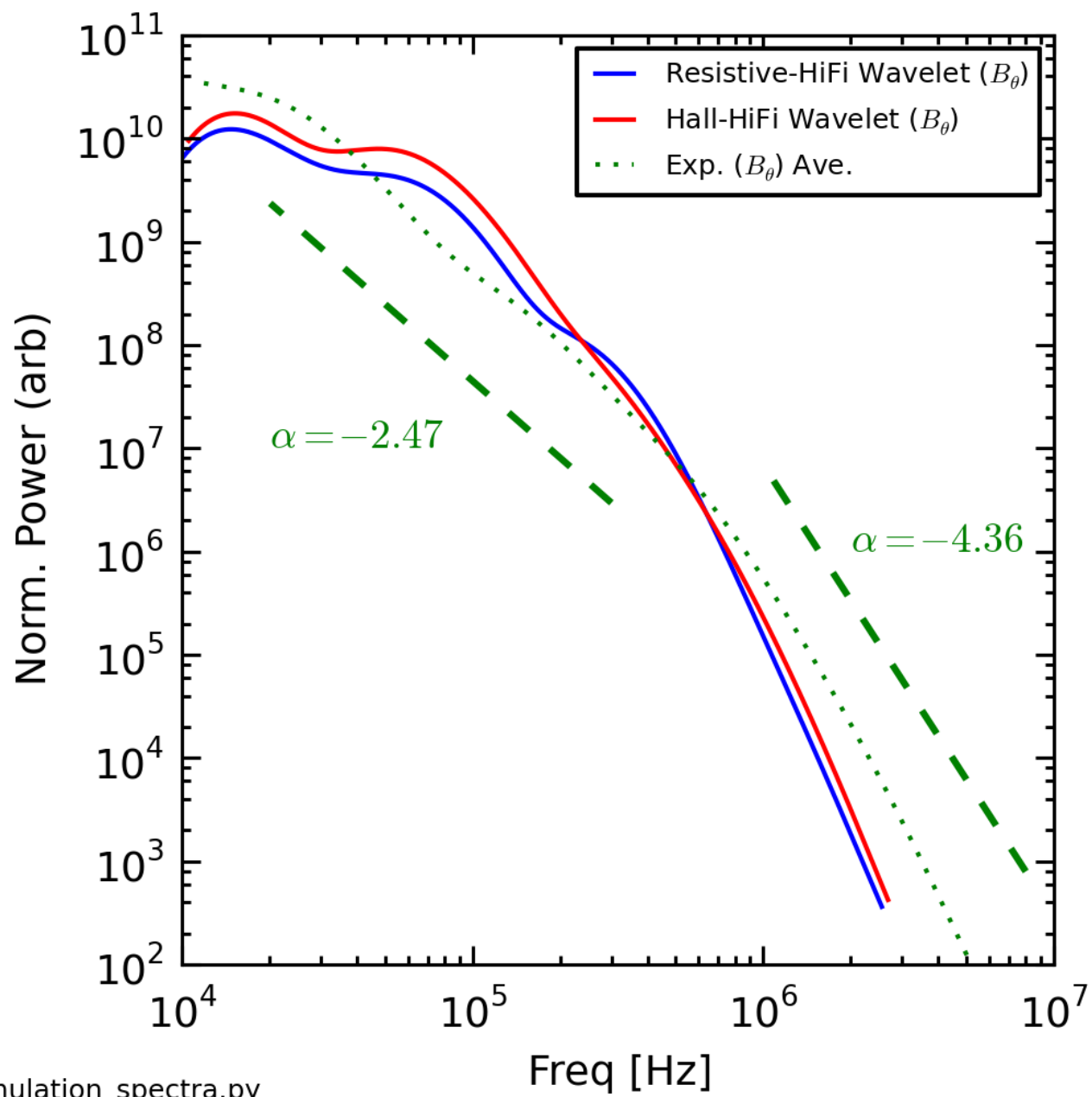
$$E(\omega) = \frac{1}{T} \left[\int B(t) e^{-i\omega t} dt \right]^2$$

- Use either FFT or Wavelets
- Convert spectrum of dB/dt to B



PPCF, Schaffner, et al (2014)





simulation_spectra.py

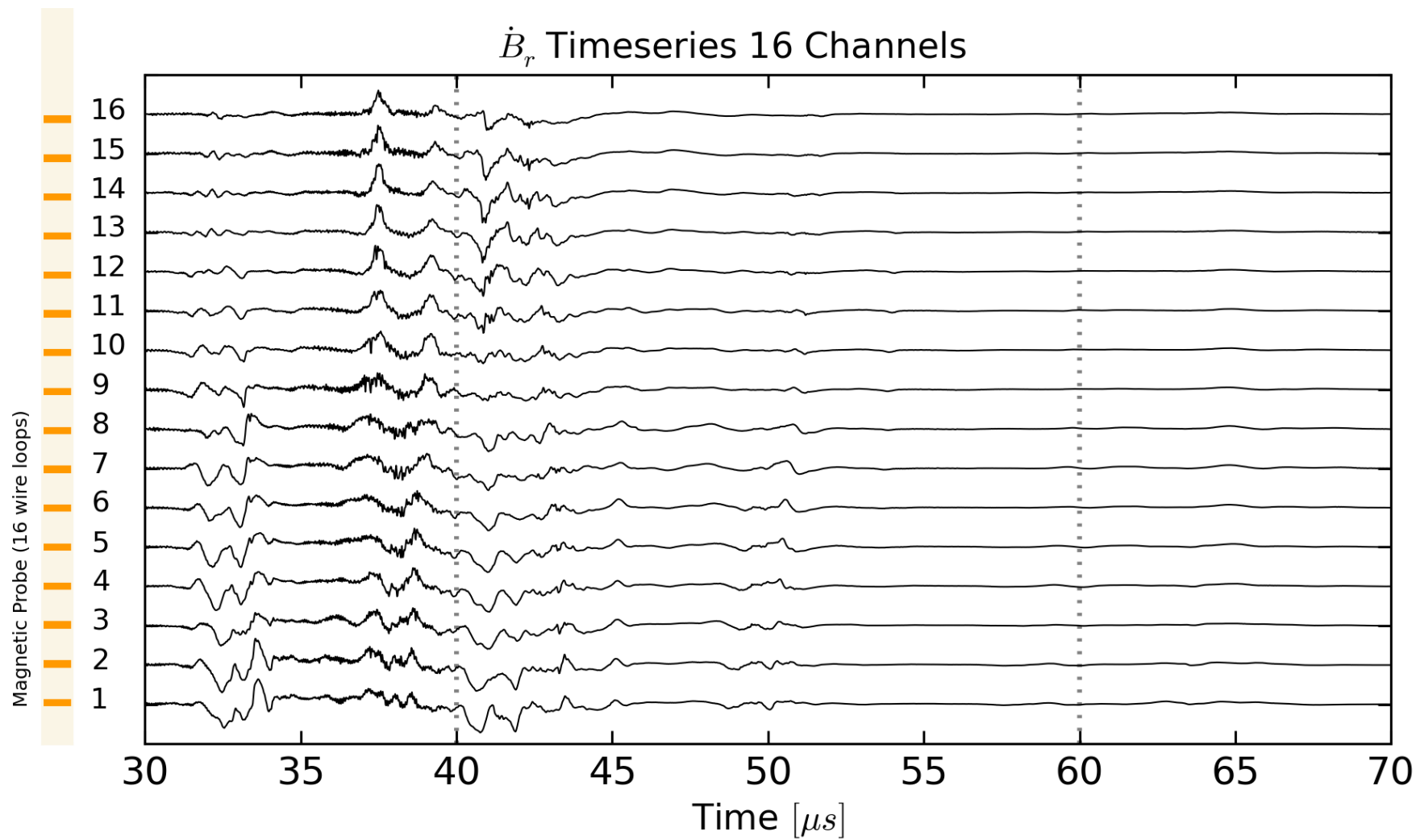
PDF of temporal increments

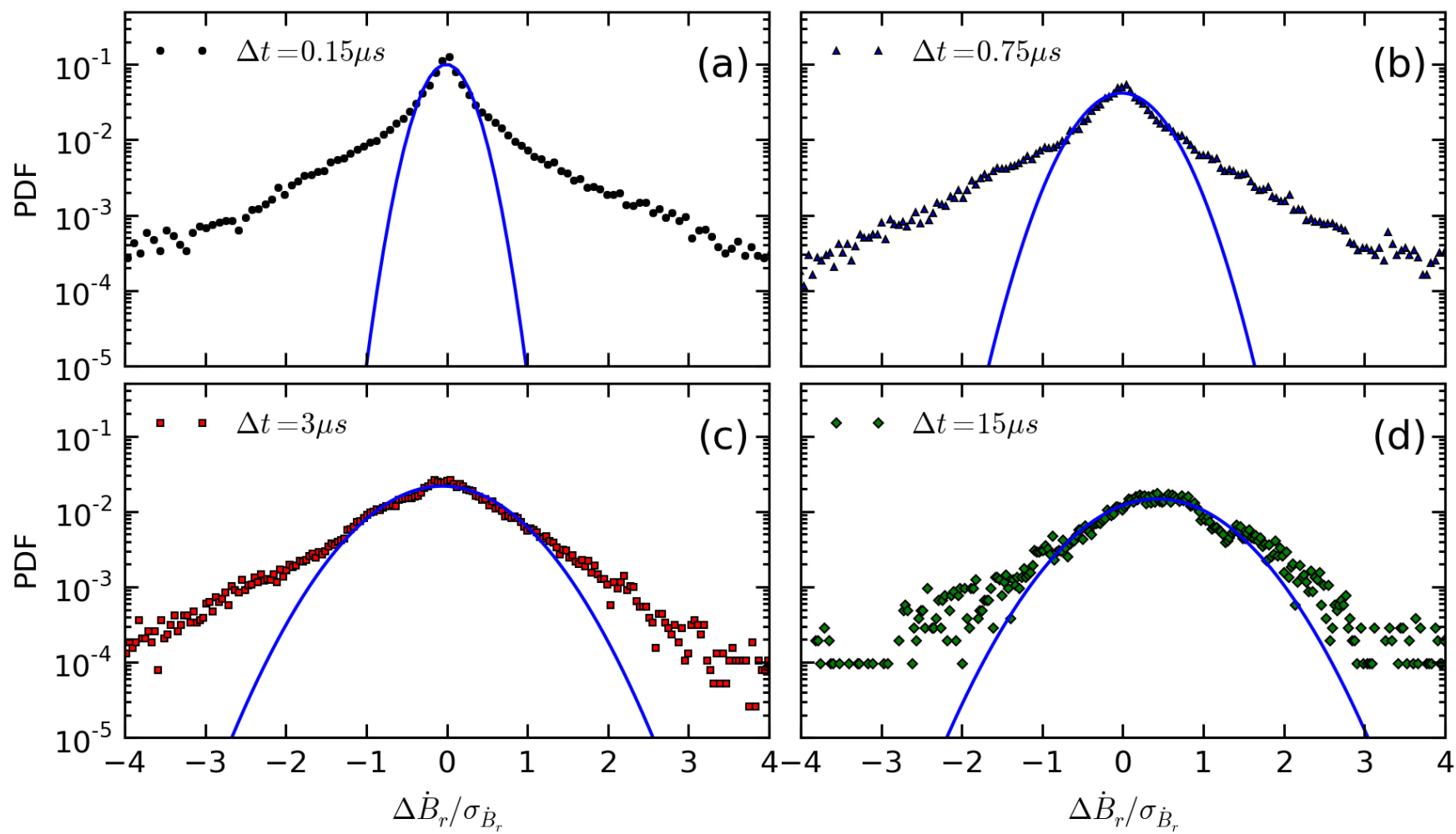
$$\Delta \mathbf{b}(t, \Delta t) = \mathbf{b}(t + \Delta t) - \mathbf{b}(t)$$

$$S^2(\Delta t) = \langle (\mathbf{b}(t + \Delta t) - \mathbf{b}(t))^2 \rangle$$

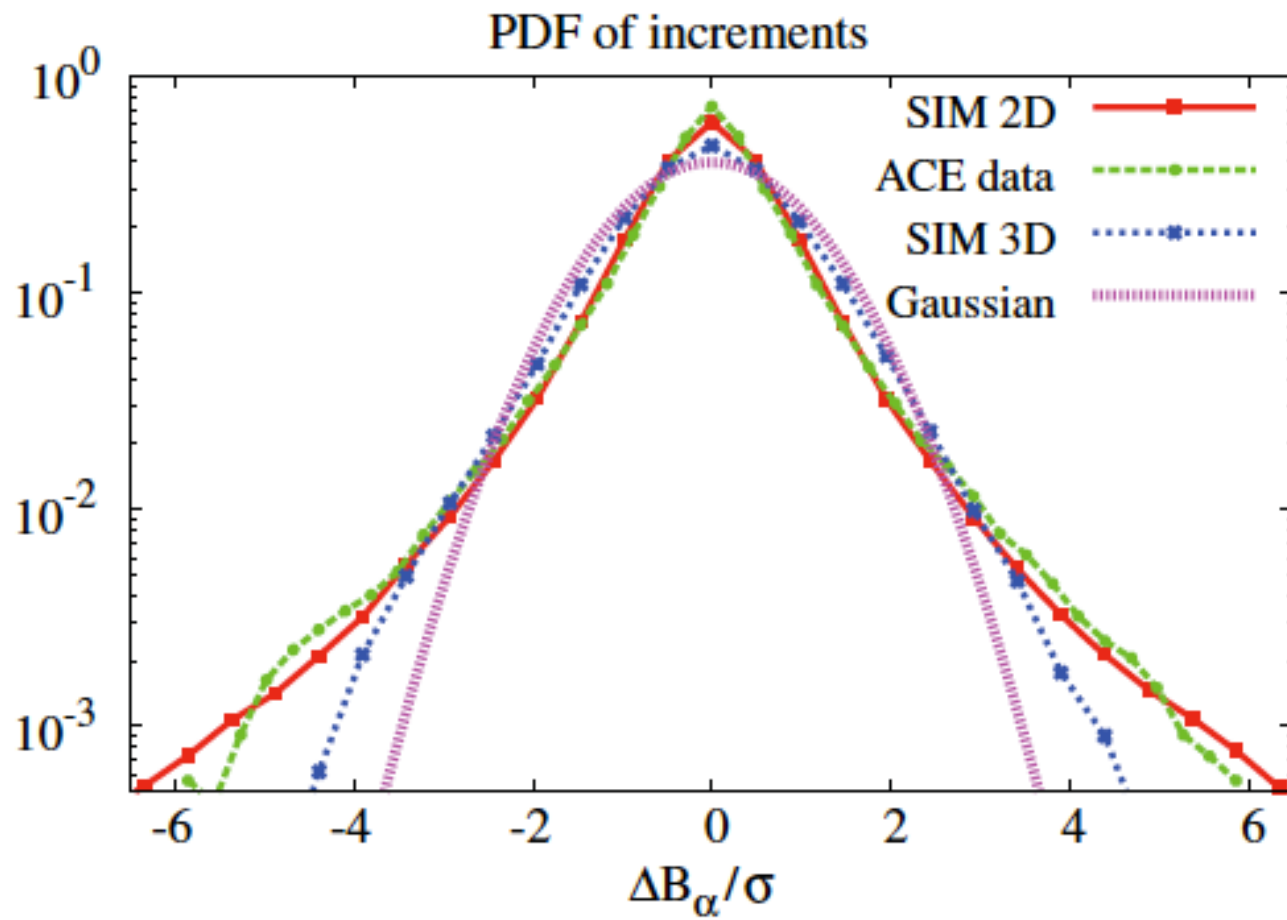
$$S^2(\Delta r) = \langle (\mathbf{b}(r + \Delta r) - \mathbf{b}(r))^2 \rangle$$

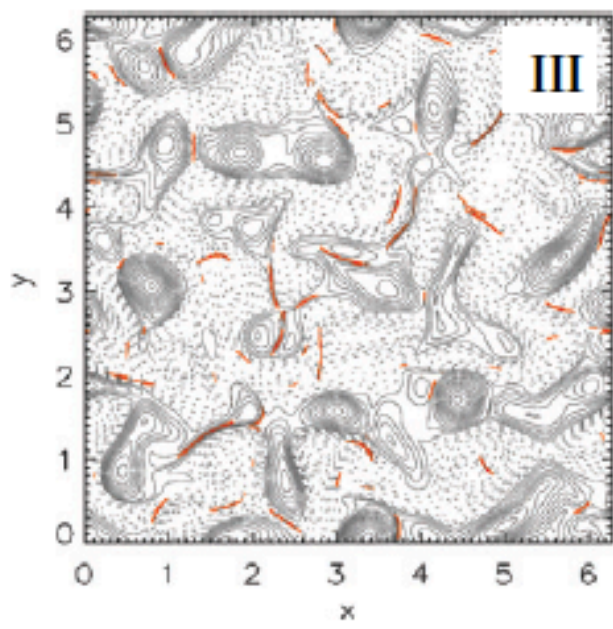
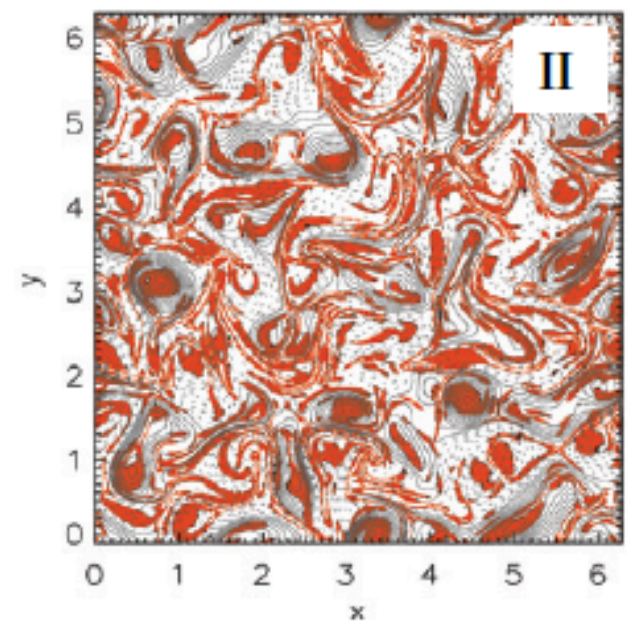
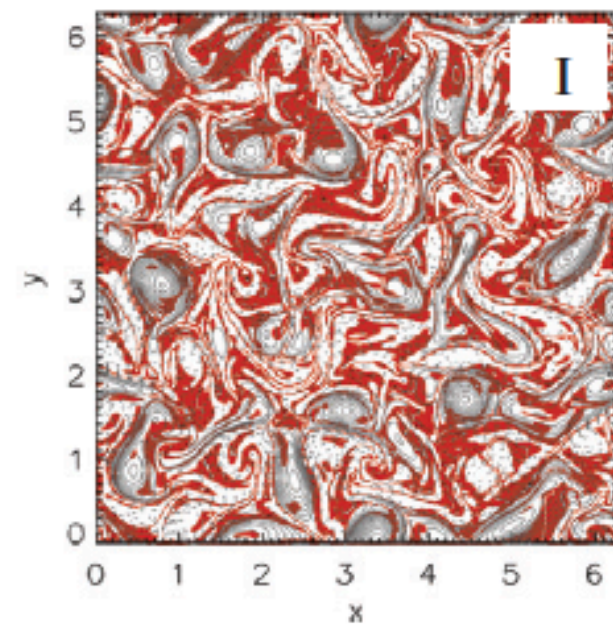
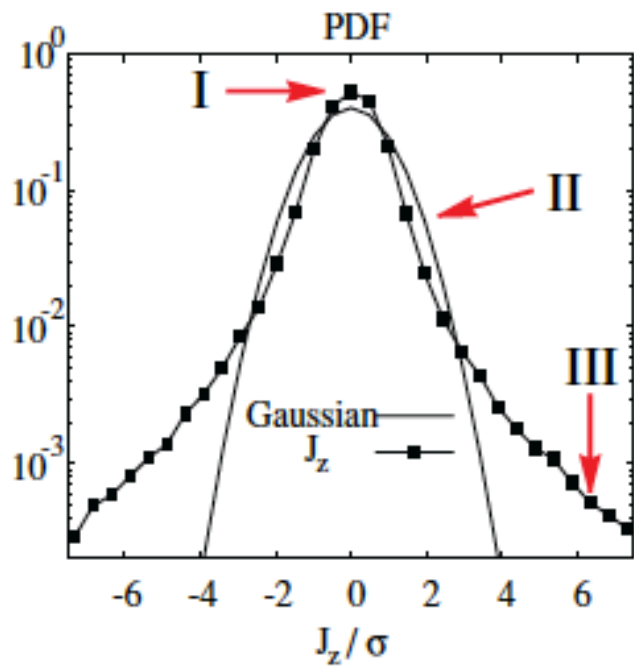
\dot{B}_r Timeseries 16 Channels





PRL, Schaffner et al (2014)



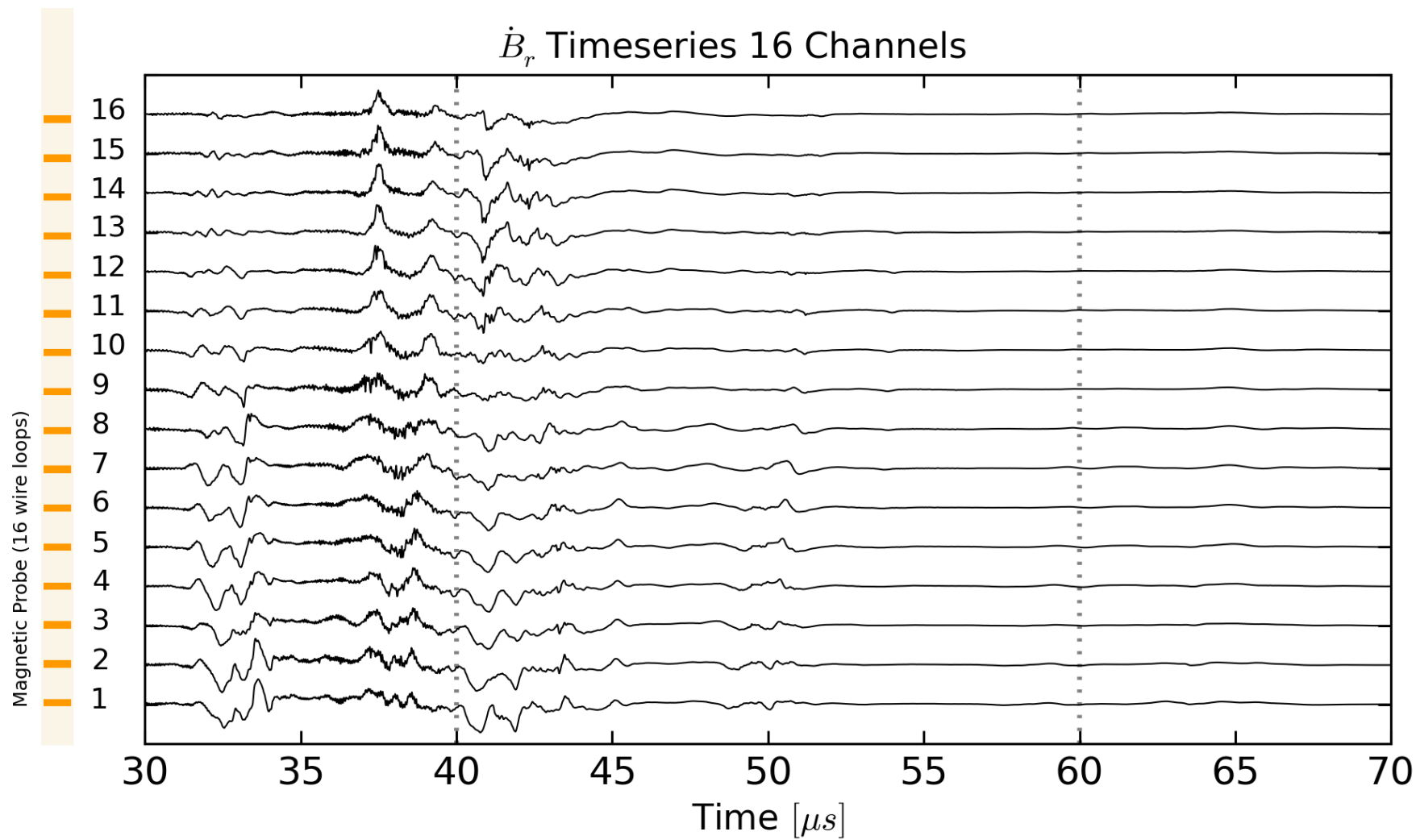


Spatial coherence function $R(\mathbf{r})$

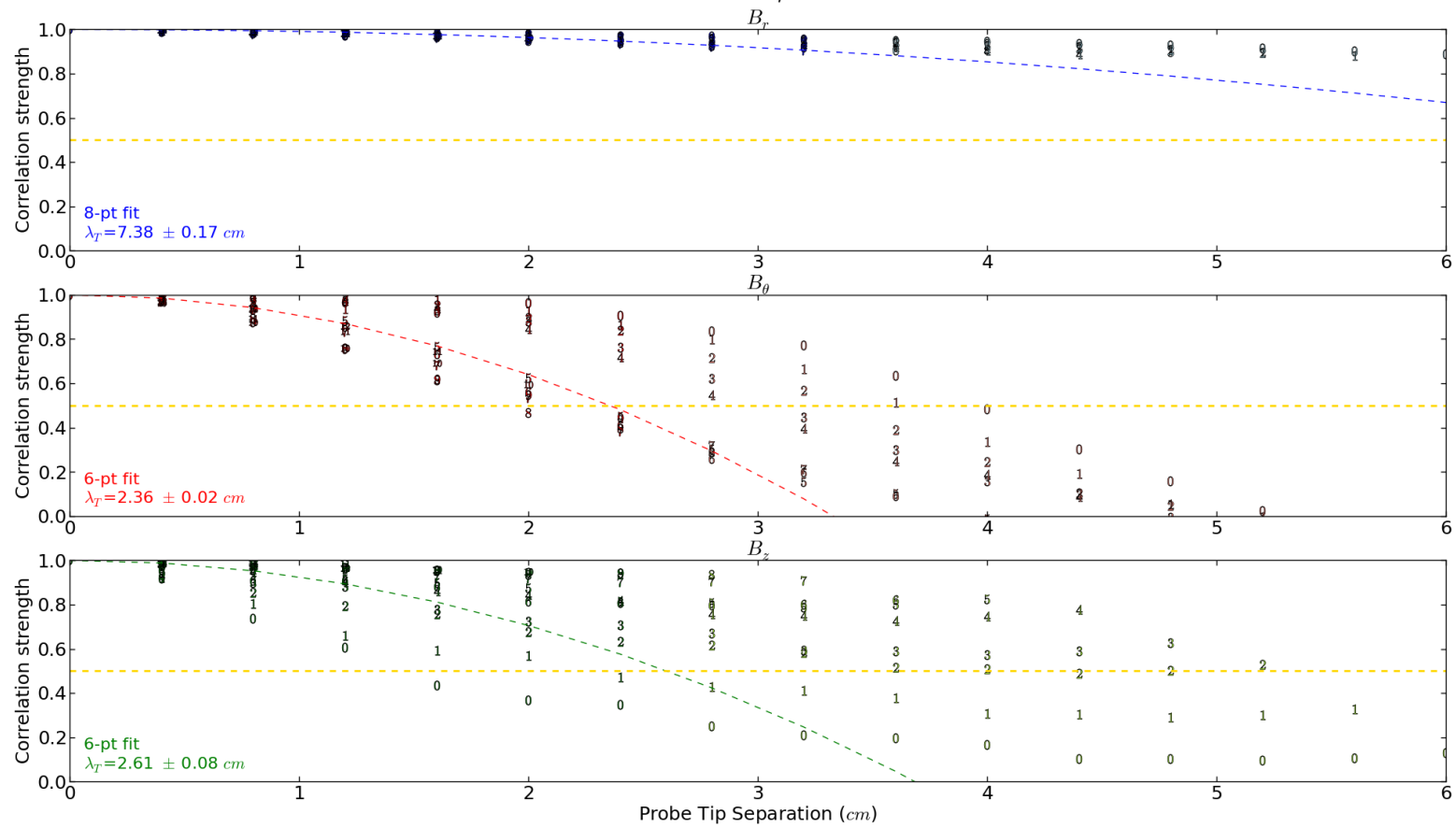
$$R_{ij}(\mathbf{r}) = \overline{b_i(\mathbf{x})b_j(\mathbf{x} + \mathbf{r})}$$

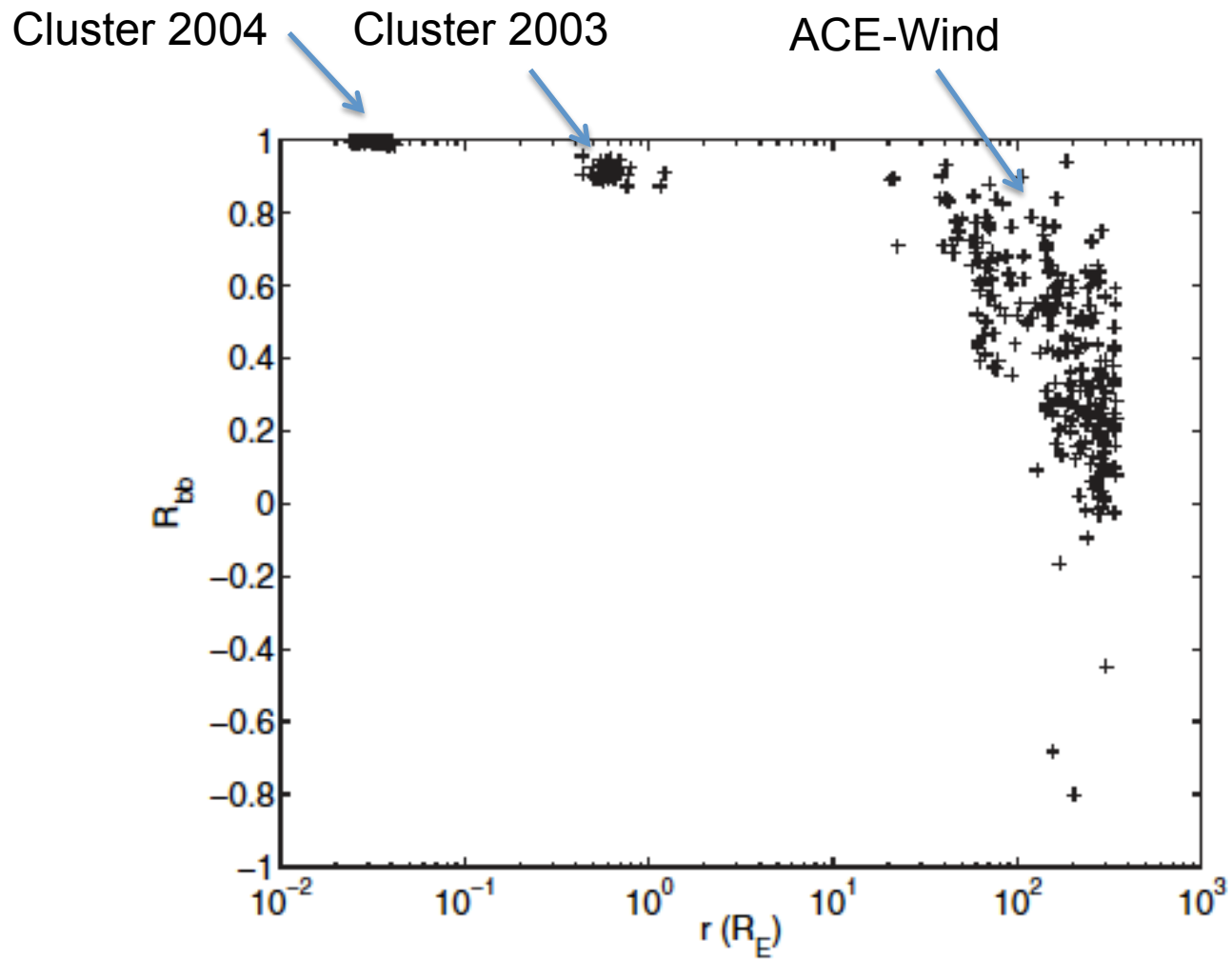
$$\frac{1}{2} \overline{b_i(\mathbf{x})b_i(\mathbf{x})} = \int_0^\infty E(k)dk$$

\dot{B}_r Timeseries 16 Channels



08/14/13, 75 shot average
40.0-60.0 μs



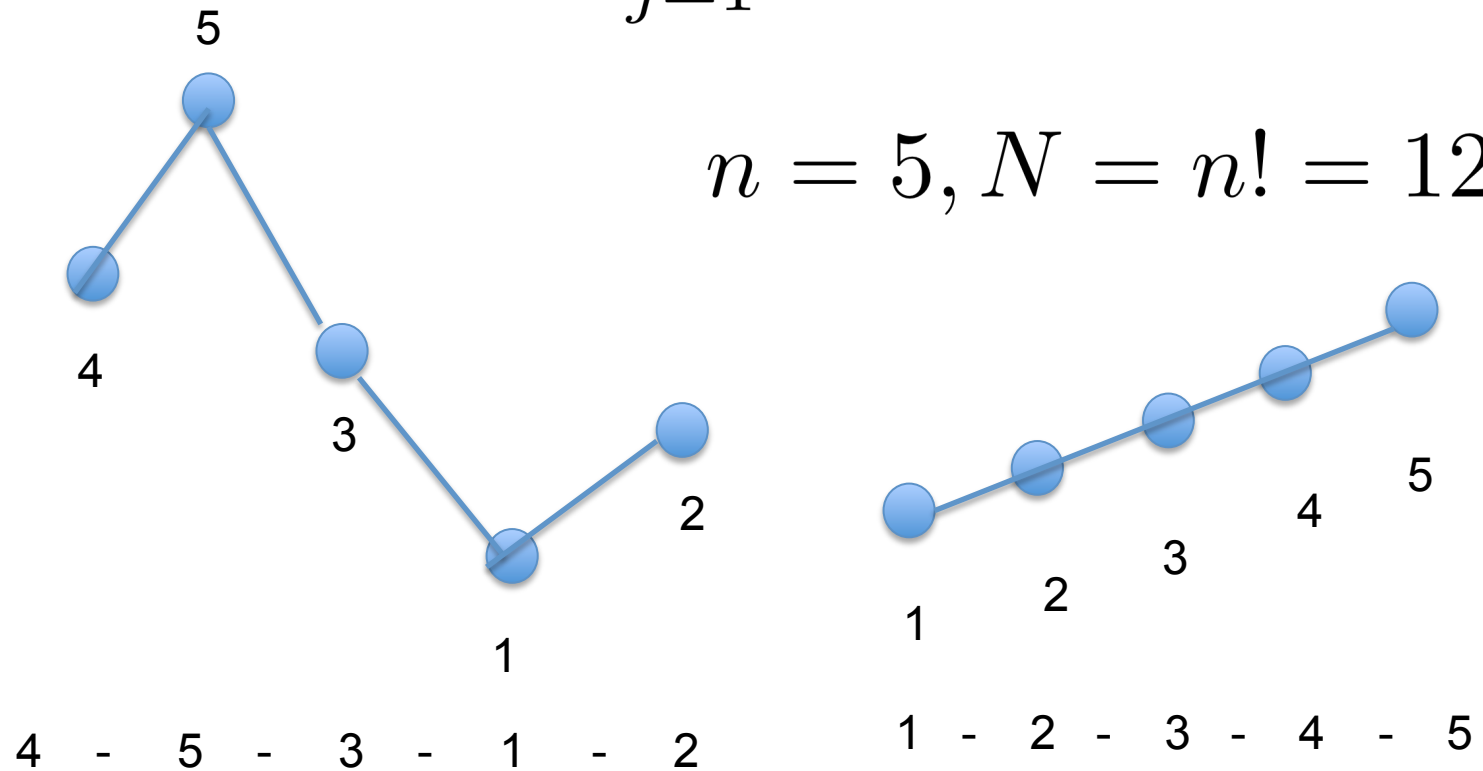


Matthaeus, 2005

Permutation Entropy

$$S[P] = - \sum_{j=1}^N p_j \ln(p_j)$$

$$n = 5, N = n! = 120$$



Permutation Entropy

$$S[P] = - \sum_{j=1}^N p_j \ln(p_j)$$

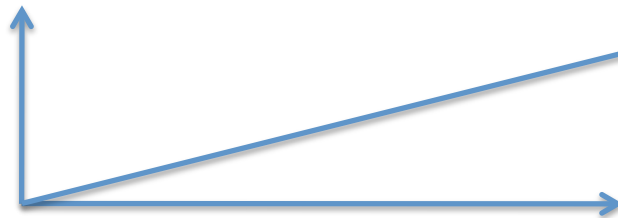
Case 1: all $N=120$ permutations equally likely...

$S = \ln(N)$... maximum



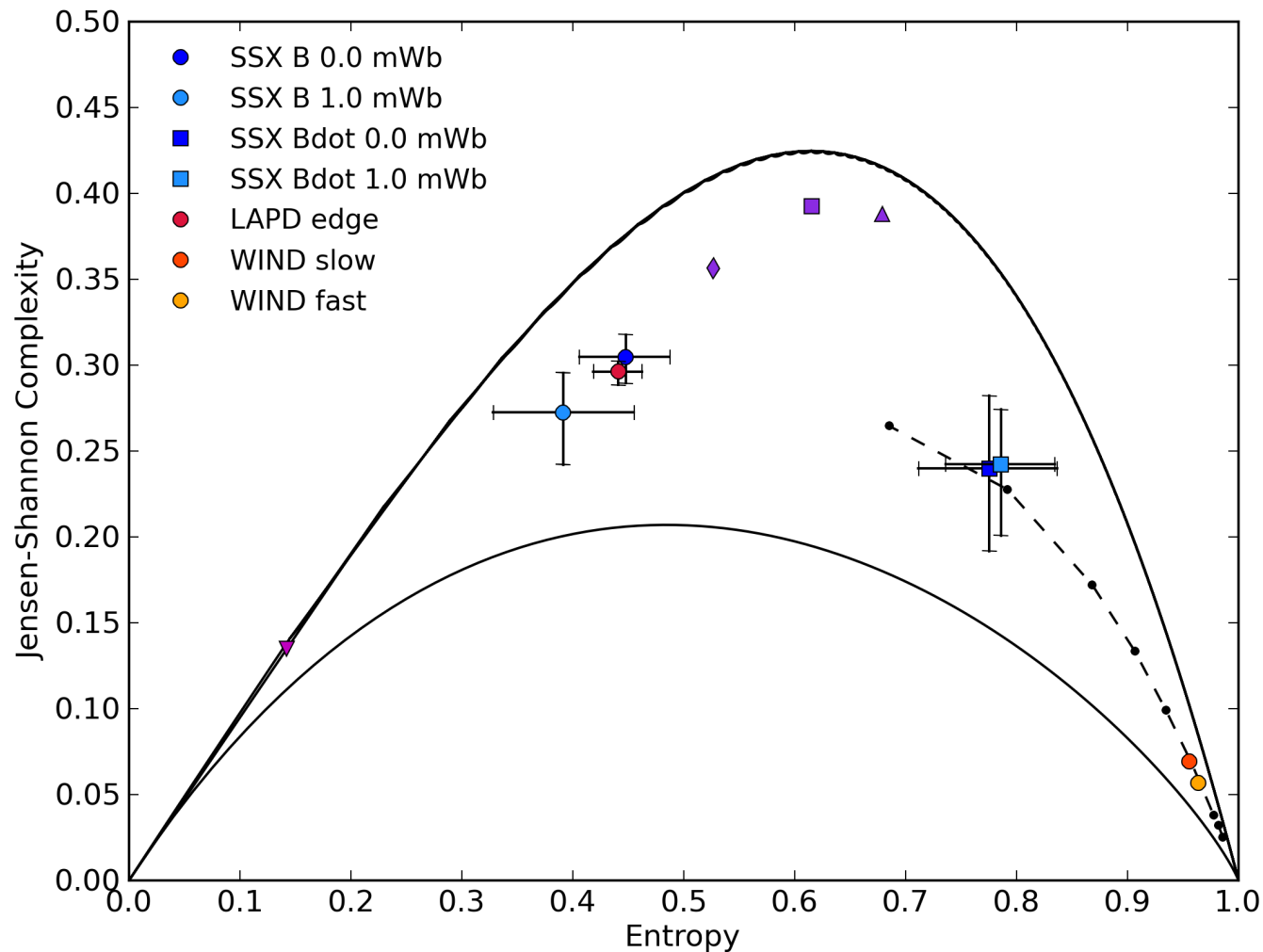
$$S = - \sum_{j=1}^N \frac{1}{N} \ln(1/N) = N \frac{1}{N} \ln(N) = \ln(N)$$

Case 2: linear ramp so only one permutation appears... $S = 0$... minimum

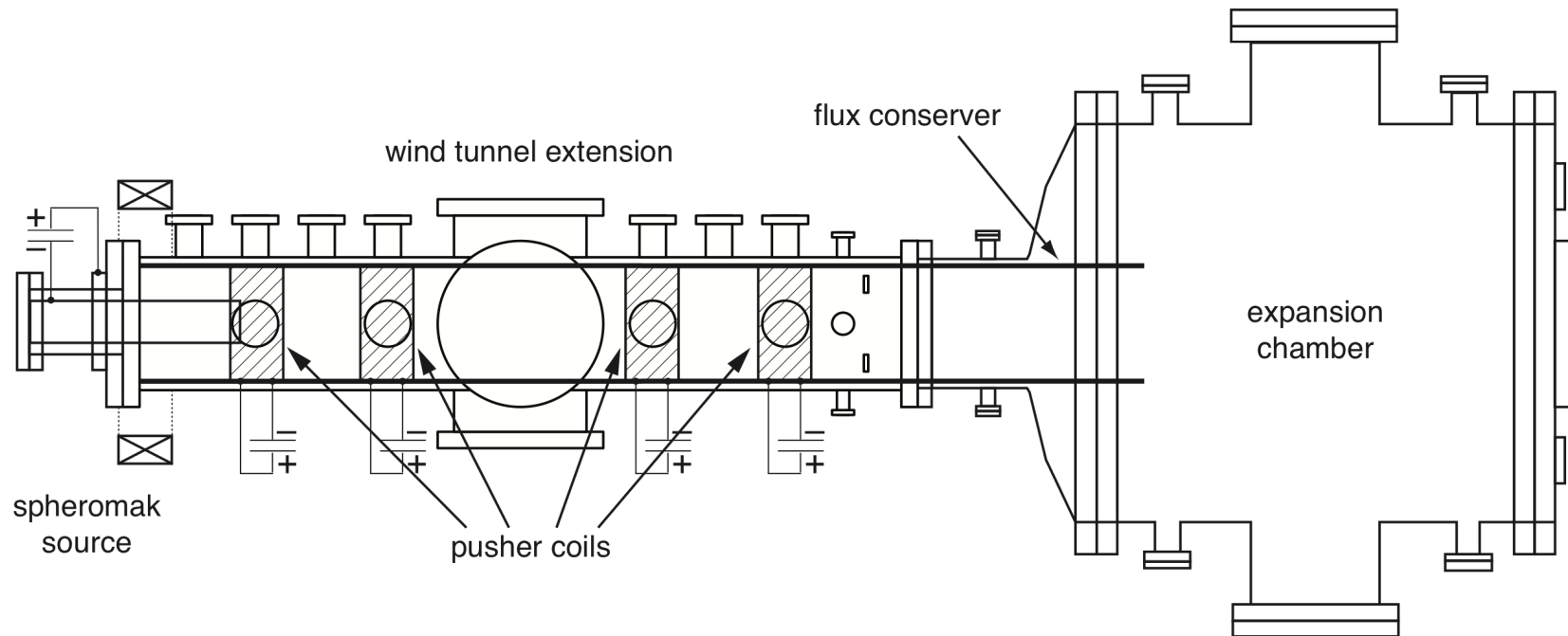


Complexity-Entropy map (SSX, solar wind, deterministic chaos)

PRE, Weck, et al (submitted 2014)

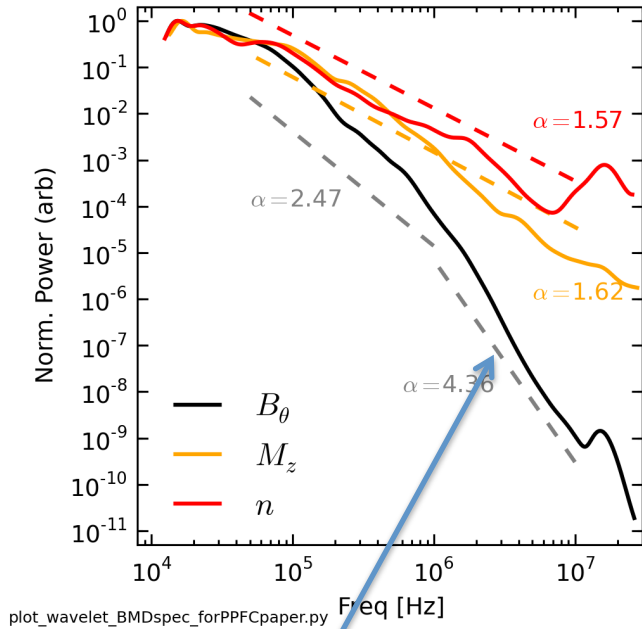


Extension and accelerator, 2015

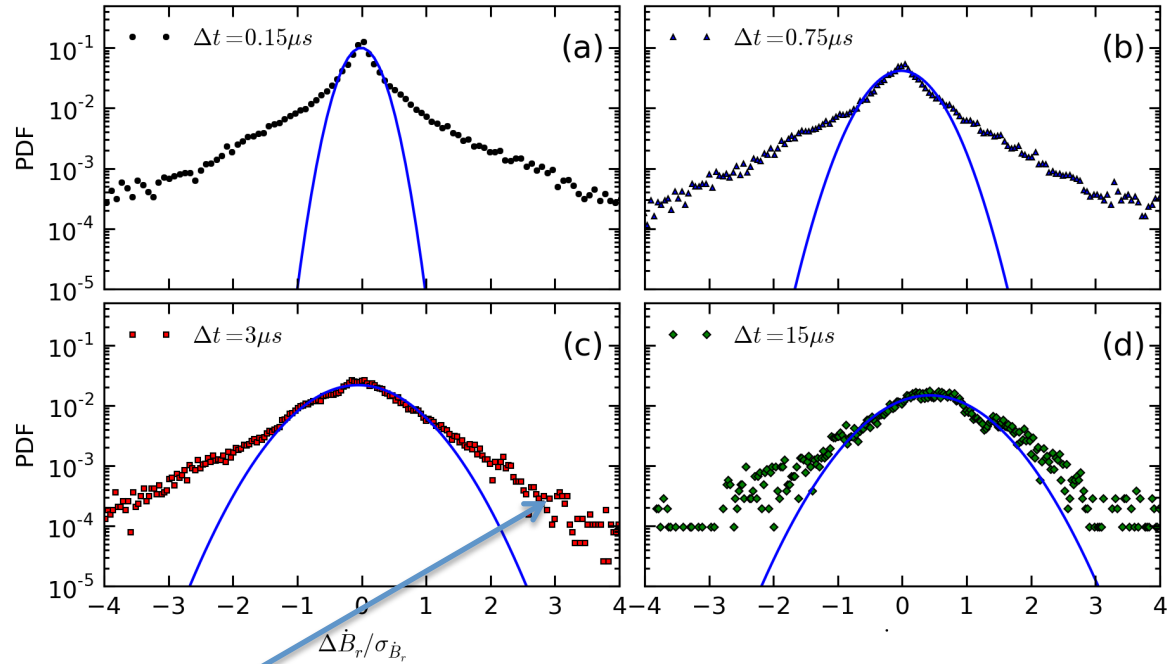


Accelerate to $v > 100 \text{ km/s}$, $M > 2$... fast camera, probes in chamber

Summary

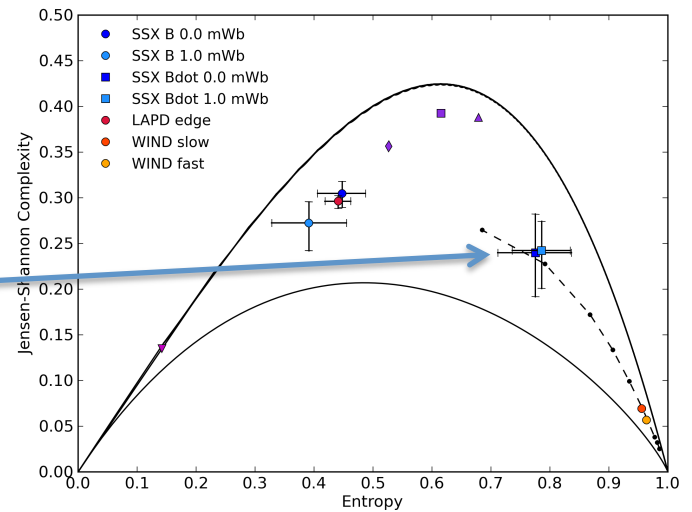


Cascade from large to small scales



Intermittency in the PDF

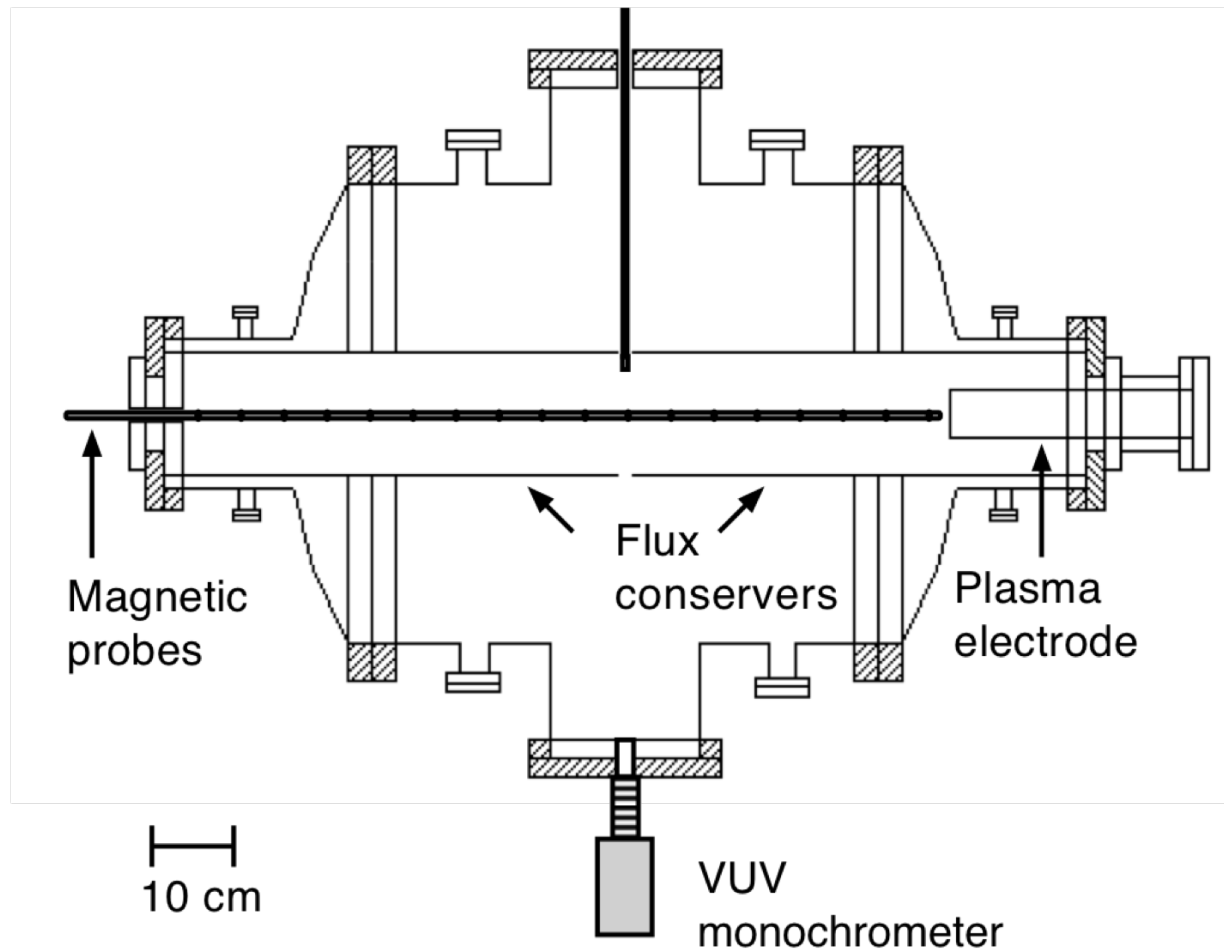
Entropy in the waveform



Thank you!

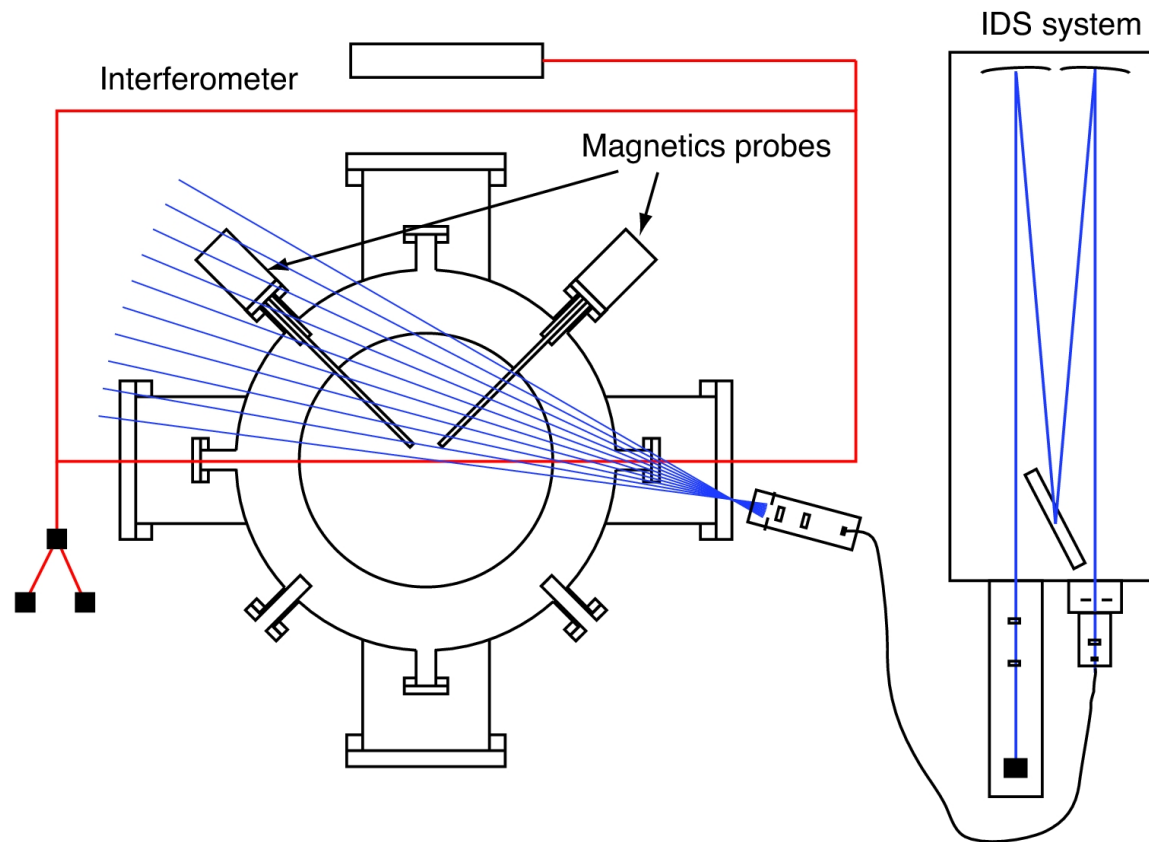
Questions?

SSX MHD wind tunnel



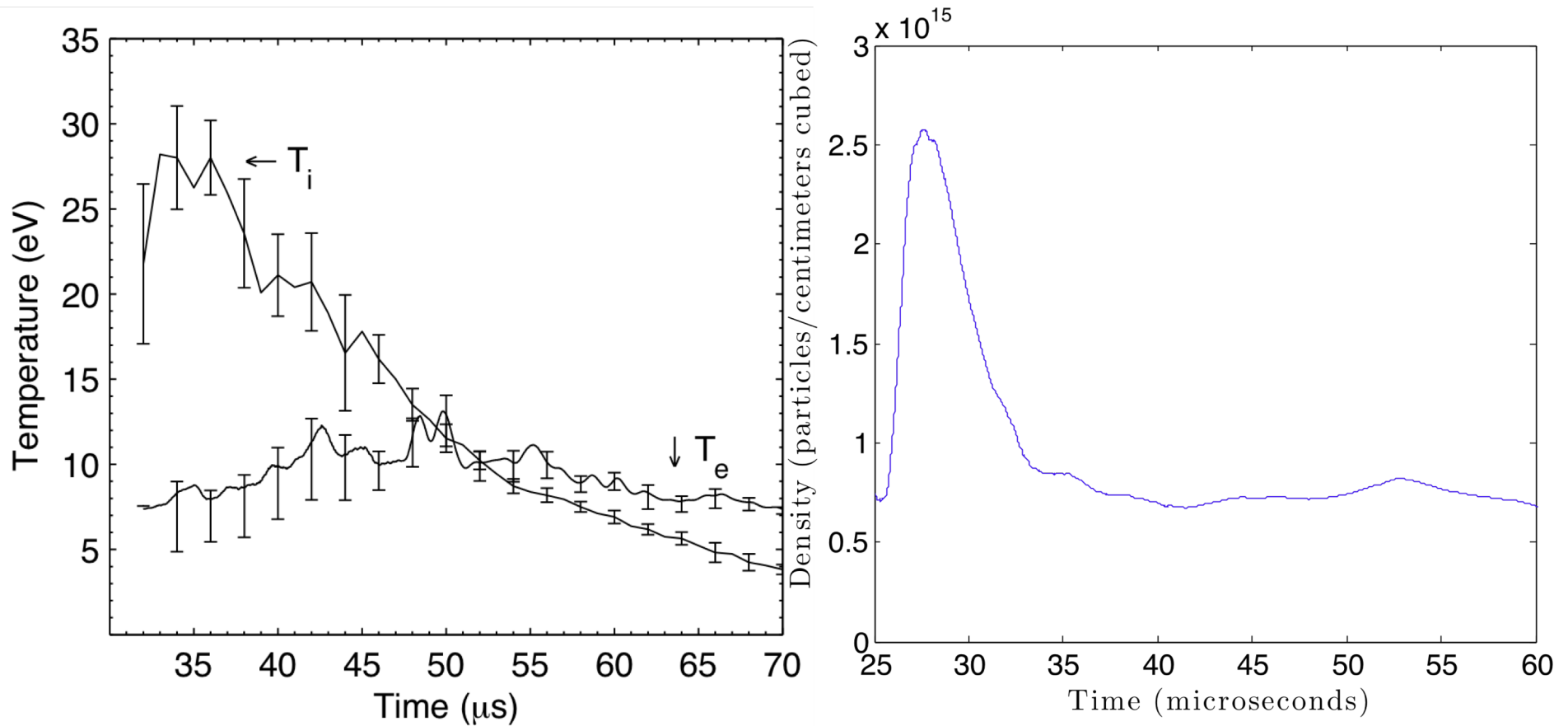
Chamber is baked and glow discharge cleaned, $P = 10^{-8}$ torr

Ion Doppler spectrometer on SSX



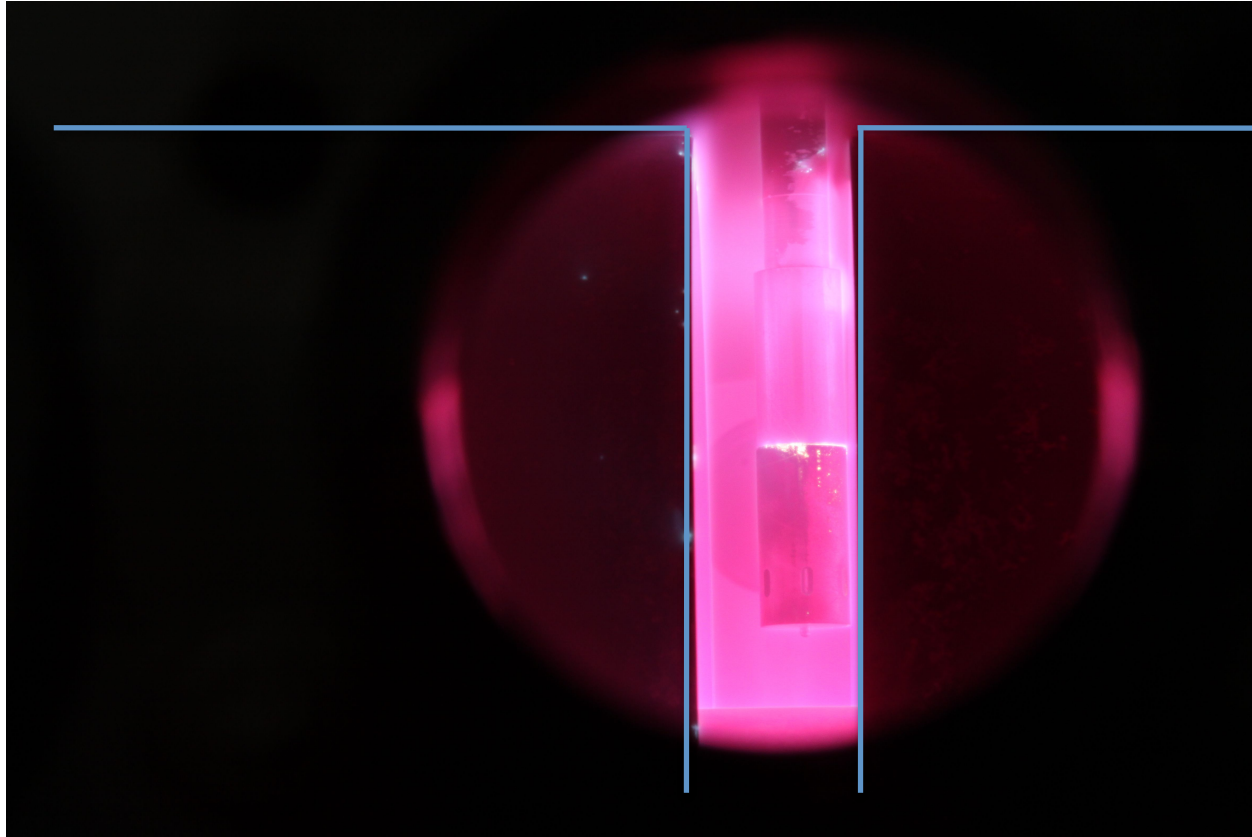
Interferometer chord and two magnetic probes also shown

Temperature and density measurements



Wind tunnel plasma is well-characterized at the midplane (line-averaged)

Mach probe measures local flow



X. Zhang