Reynolds Number Measurement from Radial Correlation Function Analysis on the SSX MHD Wind Tunnel

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Introduction

Plasma physics has become an increasingly important field of study. From understanding the universe to harnessing nuclear fusion energy, many human endeavors today require a deeper understanding of plasmas.

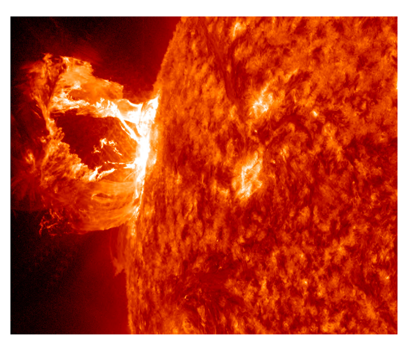

Figure 1: A coronal mass ejection event in the Sun

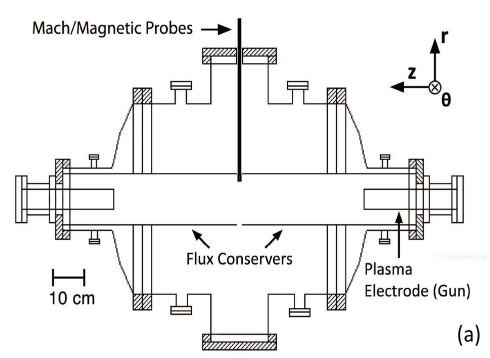

Figure 2: The SSX chamber

To this end, plasmas are created and studied at the Swarthmore Spheromak Experiment (SSX). This poster will describe the **radial correlation function**, a tool used in studying the solar wind [1], and how it is applied to SSX plasmas to calculate the Taylor microscale, a useful physical parameter. The Taylor microscale can then be used in calculating the effective magnetic Reynolds number.

Experiment

The frozen-in hypothesis states that magnetic field-lines in plasmas are convected with the mass of the plasma [2]. Hence structure in the plasma (or lack thereof) is reflected in the structure of its magnetic field.

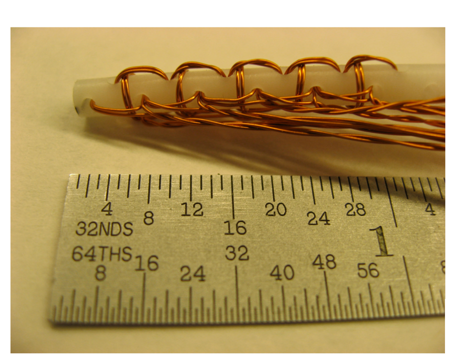

Figure 3: close-up of high-resolution magnetic probe

Magnetic structure is explored using a high-resolution magnetic probe (Fig. 3), inserted radially at the mid-plane of the wind tunnel. It measures the change in magnetic field $\dot{\boldsymbol{B}}$ in three directions at 16 points.

The SSX MHD wind tunnel employs magnetized coaxial guns. The procedure is shown in Fig. 4:

(a) Ionization of hydrogen gas
(b) Plasma acceleration through $\boldsymbol{J} \times \boldsymbol{B}$ force
(c) Poloidal field induction via stuffing field
(d) Spheromak break-off

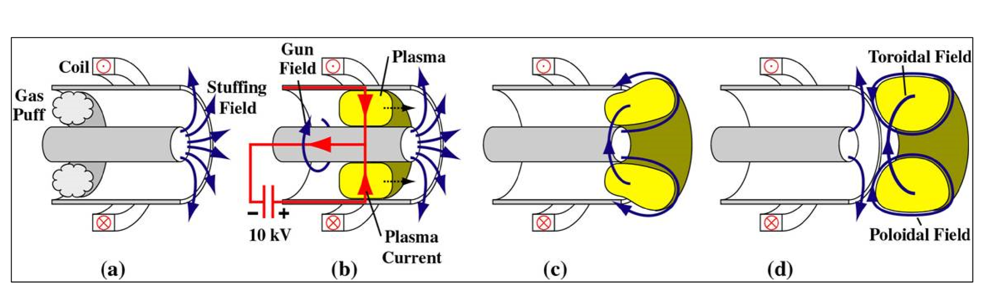

Figure 4: spheromak formation using a magnetized plasma gun

Radial Correlation Function & Analysis

Structure in the $\boldsymbol{B}$-field can be explored using the **radial correlation function**, which describes how similar a signal at a given spatial-temporal point is to itself at a different point. Given a spatial lag $\boldsymbol{r}$ and temporal lag τ, the radial correlation $R(\boldsymbol{r}, \tau)$ of a function $\boldsymbol{b}(\boldsymbol{x}, t)$ is:

$$R(\boldsymbol{r}, \tau) = \langle \boldsymbol{b}(\boldsymbol{x}, t) \cdot \boldsymbol{b}(\boldsymbol{x}+\boldsymbol{r}, t+\tau) \rangle \qquad (1)$$

A $\dot{\boldsymbol{B}}$ signal for a single shot is shown in Fig. 5. Numerical integration yields a $\boldsymbol{B}$ signal.

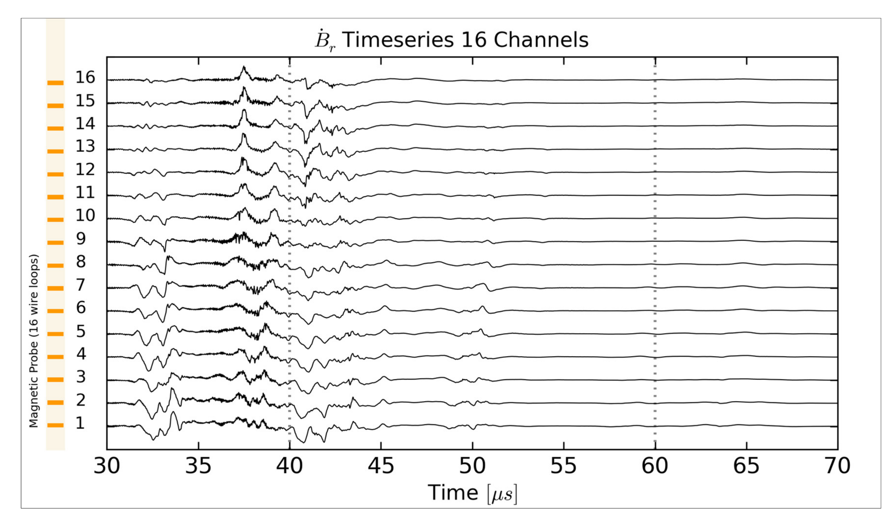

Figure 5: $\dot{\boldsymbol{B}}$ signal for one shot at all tips

To extract the maximum amount of information, *each pair* of probe tips at a separation $\boldsymbol{r}$ is used to calculate $R(\boldsymbol{r})$. Fig. 6 illustrates several probe tip pairings for $n = 1, 2, 5$.

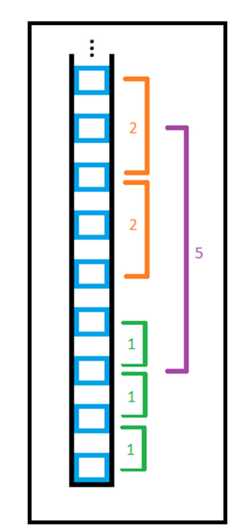

Figure 6: illustration of probe tip separations

Matthaeus' 2005 PRL illustrates using the radial correlation to measure an important physical parameter, the **Taylor microscale** λ_T, which is the scale at which dissipation commences in the plasma [1]. This relation is

$$R(\boldsymbol{r}) \cong 1 - r^2/2\lambda_T^2 \qquad (2)$$

To calculate λ_T, this equation can be fitted to a plot of $R(\boldsymbol{r})$. Fig. 7 illustrates this fitting procedure for the solar wind using data from the Cluster spacecraft (group II) [1].

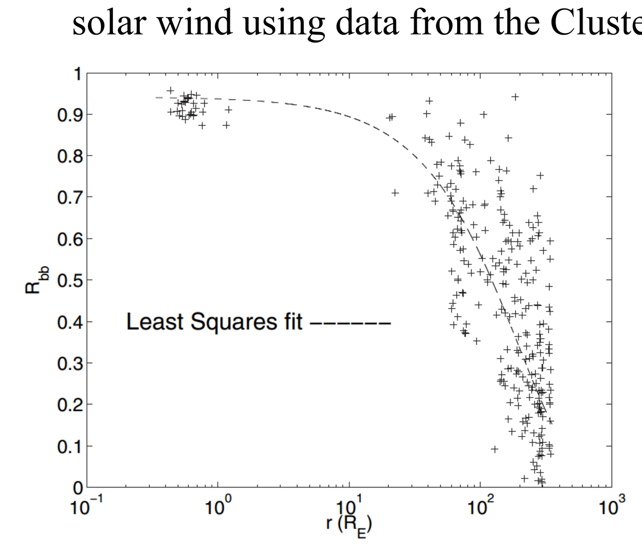

Figure 7: Fitting Eq. (2) to $R(\boldsymbol{r})$ data in the solar wind [1]

Matthaeus then calculates the effective magnetic Reynolds number R_m^{eff} using λ_T and the measured outer (length) scale λ_C:

$$R_m^{eff} = (\lambda_C/\lambda_T)^2 \qquad (3)$$

where λ_C is the longest length scale of the system.

The magnetic Reynolds number describes the effects of magnetic advection in the plasma compared to the effects of magnetic diffusion. For $R_m \ll 1$, magnetic diffusion is more important and boundary conditions largely determine magnetic structure. For $R_m \gg 1$, advection (i.e. fluid flow) is more important. Reynolds numbers give a clear indication of what mechanisms are at play within plasmas.

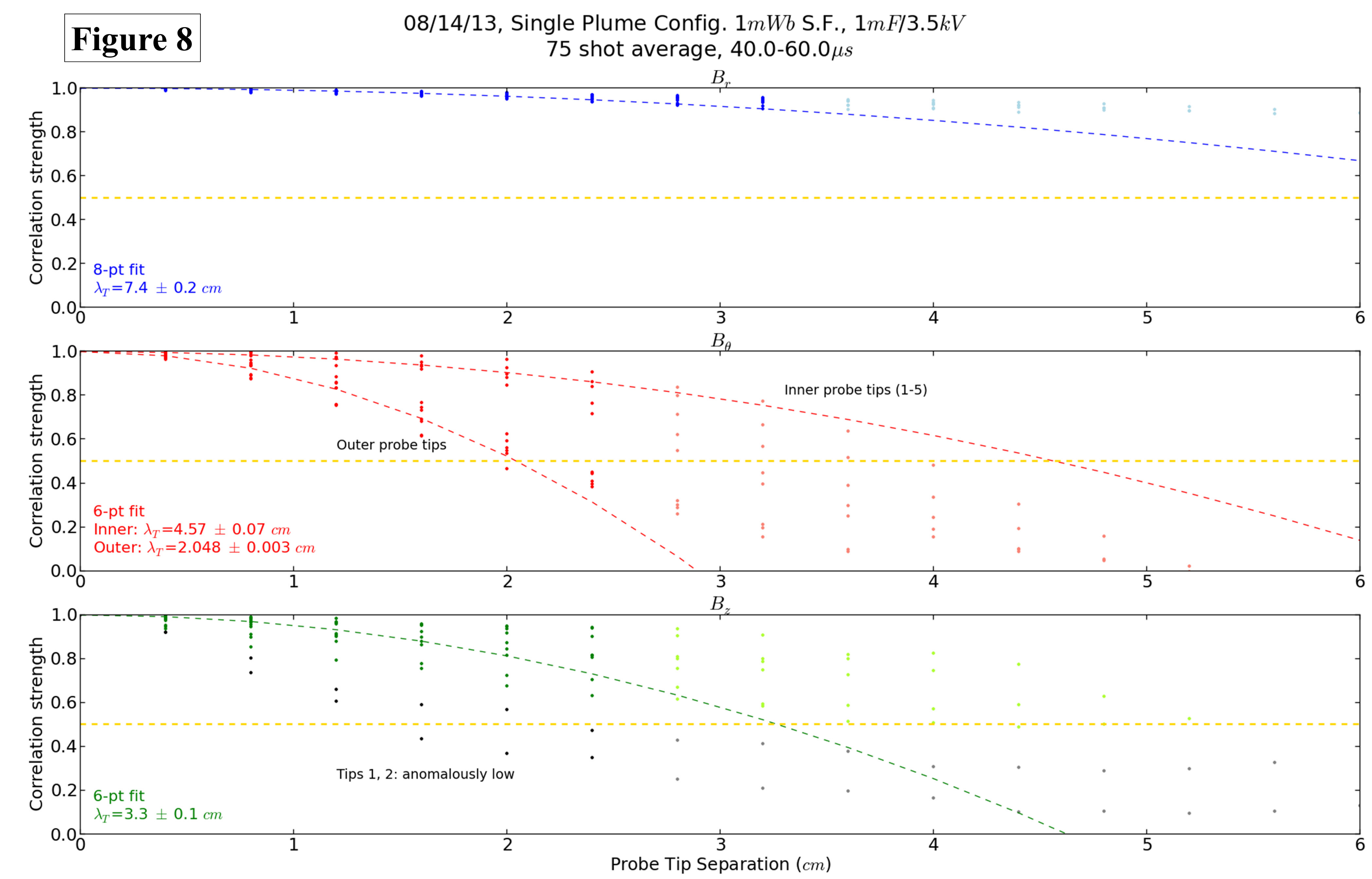

Figure 8

Discussion & Summary

Fig. 8 above shows $R(\boldsymbol{r})$ for all three components of $\boldsymbol{B}$ from 40 to 60μs. Each point represents the shot-averaged $R(\boldsymbol{r})$ value measured between probe tip n and tip $n{+}r$. Eq. (2) produces the dotted it lines.

The B_θ plot exhibits a bifurcation of $R(\boldsymbol{r})$ values. The larger $R(\boldsymbol{r})$ are correlations taken with the innermost probe tips; this robust pattern persists after a 75-shot average, justifying the making of two fits.

We take λ_C to be the radius of the flux conserver because this is the largest scale for the radial correlation measurement. We use the outer probe tip fit from the B_θ as we seek the smallest upper bound for the dissipation scale. Thus the best estimate of the effective magnetic Reynolds number is

$$R_m^{eff} = (7.7cm/2.05cm)^2 \cong \mathbf{14}$$

This technique extracts the magnetic Reynolds number purely from an analysis of fluctuations. The result above can thus be compared to R_m^{eff} values calculated using other methods.

Literature cited

[1] W. H. Matthaeus, S. Dasso, J. M. Weygand, L. J. Milano, C. W. Smith, and M. G. Kivelson, *Spatial correlation of solar-wind turbulence from two-point measurements*, Phys. Rev. Lett. **95**, 231101 (2005).

[2] P. M. Bellan. *Spheromaks: A Practical Application of Magnetohydrodynamic Dynamos and Plasma Self-organization*. London: Imperial College, 2000. Print.

Acknowledgments

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For further information

Please contact **awan1@swarthmore.edu**. More information on SSX can be obtained at **http://plasma.swarthmore.edu/SSX/index.html**