A large-scale horizontal-vertical illusion produced with small objects separated in depth

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Abstract

Two experiments (total N = 81) we conducted to investigate the basis for the large-scale horizontal-vertical illusion (HVI), which is typically measured as 15-20% and has previously been linked to the presence of a ground plane. In a preliminary experiment, vertical rods of similar angular extents that were either large (4.5-7.5 m) and far, or small (0.9-1.5 m) and near, were matched to horizontal extents in a virtual environment by adjustment of horizontal gaps or rods. Large/far objects showed a larger HVI (~ 13%) than small objects (~7%), as has been shown before, but the horizontal gap normally used to measure the large-scale HVI was not the source of the larger bias. In the second experiment, it was found that simply separating the comparison rod in depth from the vertical rod (thus forcing an evaluation of size at a distance) was sufficient to produce a large HVI (17%) even with small rods. The results are interpreted in light of evidence that the large-scale HVI is dependent on ground plane orientation and may be related to differential angular expansion in the visual coding of elevation and azimuth.

Public significance statement

This study shows that the visual perception of the vertical and horizontal sizes of objects in 3D space differ. This difference is much larger when the sizes to be compared are not at the same distance from the observer, so that relative size judgments cannot be deduced from their projected size. These large distortions in perceived size are quantitatively consistent with models of perception that emphasize systematic angular coding bias as a perceptual strategy for enhancing precision for action.

A large-scale horizontal-vertical illusion produced with small objects separated in depth

The horizontal-vertical illusion (HVI) normally refers to the observation that vertical lines generally appear 2% -10% longer than horizontal lines (Fick, 1851; see Landwehr, 2016). In this paper we consider the basis for a much larger HVI that has consistently been reported for large-scale objects (Chapanis & Mankin, 1967; Higashiyama, 1992, 1996; Klein, Li & Durgin, 2016; Yang, Dixon & Proffitt, 1999). This *large-scale* HVI has a magnitude of 15%-25% when large objects (e.g., 4 m tall or more) are used as stimuli. It is presently unclear exactly why large objects show such a large effect. Theories concerning gravity (e.g., Yang et al.) and the use of visual ground plane in scaling size and distance (Klein et al.) have both been proposed. Here we will show that the effect may be the result of a 3D size evaluation process that can also be applied to smaller objects on the ground plane. Additionally, one of the issues we will address is that, for practical reasons, large-scale HVIs are usually measured by adjusting the size of an empty interval to match a vertical extent. We will show that this method does not account for the large magnitude of the effect.

The most common representation of the normal 2D HVI is the inverted T figure, but this figure combines two different effects: The vertical line differs not only in its orientation, but it also bisects the horizontal line, and this is known to magnify the apparent size difference. Rotating the T figure by 90° essentially eliminates all bias (e.g., E. O. Cormack & R. H. Cormack, 1974). When an L-shaped figure is used, the magnitude of the illusion is typically about 3-6% (Künnapas, 1955). That is, the vertical and horizontal lines will appear perceptually equal when the horizontal line in the figure is about 5% longer than the vertical line. Many other kinds of manipulation can also affect the magnitude of the effect (e.g., Armstrong & Marks, 1997; Avery & Day, 1969) including simply separating the lines so that they no longer form a unit (e.g., Teghtsoonian, 1972; Cai, Wang, Song & Li, in press). This simple manipulation tends to produce a somewhat larger illusion of 7-10%.

In a study of the 2D HVI illusion, with separated lines, P. A. Williams and Enns (1994) identified two additive contextual components that affected the magnitude of the HVI, though neither accounted for the entire effect. On the one hand, the aspect ratio of the frame surrounding the lines had a strong impact on the apparent match point (see also Prinzmetal & Gettleman, 1994). On the other hand, presenting a pictorial depiction of a receding plane in the background (either a horizontal receding plane or a vertical receding plane) produced a separate, additive effect on the magnitude of the HVI. On average, Williams and Enns measured an HVI of 6-8%, which increased to 10% or decreased to 4% depending on the contributions of their two manipulations. The main (6-8%) component of the 2D illusion might be predicted by statistical properties of the environment (Howe & Purves, 2002; Zhu & Ma, 2017). Even with the pictorial depth plane and the frame encouraging a larger HVI, Williams and Enns measured HVI magnitudes of just 10%. Nonetheless, the observation of an effect of the ground plane seems potentially relevant to understanding the large-scale HVI, which is normally evaluated with objects on a real ground plane.

Indeed, in their recent investigation of the large-scale HVI, Klein et al. (2016) found evidence of two different contributions to the large-scale HVI, the larger of which was the orientation of the ground plane. They contrasted retinotopic components and allocentric components of the illusion by using re-oriented observers (observers lying on their sides at eye level outdoors) or re-oriented virtual environments (using head-mounted immersive simulated visual environments that had been rotated by 90°). Under these conditions, they found a relatively small (~6%) component that was fixed to the orientation of the head, and a larger (~17%), allocentric component of the HVI that depended on the orientation of the ground plane, with the result that the orientation of the ground plane determined the overall sign of the effect. The smaller, retinotopic component is consistent in magnitude with the typical size of the 2D HVI, whereas the component associated with ground-plane is sufficient to explain the typical magnitude of the large-scale HVI. The ground plane (including the horizon it defines) is widely recognized as an important source of information relevant to scaling perceived distance (e.g., Messing & Durgin, 2005; Ooi, Wu & He, 2001; Wallach & O'Leary, 1982; M. J. C. Williams & Durgin, 2015) and size (Sedgwick, 1973; Wraga, 1999). The large-scale effect might therefore be related to the computation of 3D object sizes (rather than merely comparing relative 2D size within a plane).

However, the large-scale HVI procedure used in most experiments (common to Chapanis & Mankin, 1967, Klein et al., 2016, and Yang et al., 1999) involves comparing a vertical *object*, with a horizontal *gap* between that object and another object (such as a person). This detail is a matter of practicality: Adjusting the size of a physical gap is simpler to implement in a large-scale 3D environment. Given that dis-uniting the vertical and horizontal components of a typical 2D HVI comparison increases the HVI, it seems important to establish whether or not the very large magnitude of the large-scale HVI is due primarily to this asymmetry. Therefore, prior to conducting a test of the ground-plane hypothesis, we sought to first establish whether an L-shape configuration of poles could produce a large-scale HVI.

Experiment 1: Preliminary test of horizontal gap vs. pole in large-scale HVI

Because we sought to use a form of presentation (an immersive 3D wall) as a means of measuring the large-scale HVI, that has not been used previously for this purpose, we combined our test of the "gap" hypothesis with a simple replication of the large-scale HVI. That is, we crossed the type of horizontal comparison used (a gap or a pole) with the size of the vertical poles used. If the large-scale effect depended on the use of a gap, we should expect to see an interaction between the scale of the vertical pole and the type of extent (gap or pole) of the horizontal comparison interval.

Methods

Participants. Thirty-six undergraduates from Swarthmore College participated in this experiment for payment. All the participants had normal or corrected to normal vision. The experimental procedures were approved by the local research ethics committee.

Apparatus. The experiment was conducted in a one-wall stereoscopic virtual environment, using a PROPixx projector (resolution: 1920 x 1080 @ 120 Hz; VPixx Technologies Inc.) and a large, polarization-preserving, back projection screen (with a width of 2.56 m). An active 3D circular polarizing filter alternated polarization so that odd frames and even frames contained orthogonally polarized signals, which were segregated to the two eyes by corresponding passive polarized lenses, producing a strong immersive 3D effect. Each observer's inter-pupillary distance (IPD, measured with Shin-Nippon PD-82) was used in the simulation. During the experiment, the participants sat on a chair at a viewing distance of about 1.2 m from the screen producing a FOV of roughly 90°

horizontally and 62° vertically. Their head position, which was monitored by a motion tracking system (PPT-E, Worldviz Co.), was used to update the display, which was programmed and rendered using Vizard. Head excursions were not recorded, but were probably only a few centimeters during most trials of the experimental procedure. Note that, contrary to popular wisdom, stereoscopic information is useful for perceiving depth and distance along a ground plane for dozens of meters (Allison, Gillam and Vecellio, 2009). The purpose of head-tracking was to maintain an accurate geometry.

Stimuli and procedure. The virtual environment, shown in Figure 1, was composed of a large ground plane with a detailed (random noise) ground texture and a cloudy sky. A large plant was simulated 2.5 m from the participants (in the lower left of the visual field) to enhance immersion. In the gap condition (Figure 1, left), a vertical pole was positioned directly ahead of the participant, with a shorter vertical pole to the right of it. The participant's task was to adjust the position of the shorter pole (leftward or rightward) using a joystick so as to set the horizontal distance between the two poles so that it appeared to equal the height of the vertical pole. In the horizontal pole condition (Figure 1, right), participants adjusted the size of a horizontal pole to match the size of the vertical one. To study the effect of apparent size, the HVI tasks were conducted in two scales. In the large scale (Figure 1, top, the vertical pole was either 4.5, 6 or 7.5 m tall, viewed at either 20, 25, or 30 m. In the small (20%) scale (Figure 1, bottom), pole heights of 0.9, 1.2, or 1.5 m were tested at distances of 4, 5, and 6 m. Thus, the retinal size of the poles was approximately matched across scales. Note that foreshortening of the poles by viewing angle was similar for the vertical poles in the small and large conditions, which extended either mostly below, or mostly above the line of sight, respectively.

Design. The two types of horizontal extent (gap or pole) were tested between subjects; the two object scales (small, large) were blocked and tested within subject with order (large-scale block of trials first or small-scale first) varied between subjects. In each block, the nine combinations of vertical pole height and viewing distance were tested twice in random order, with the horizontal distance shown both initially much shorter and initially much longer than the vertical height to reduce anchoring effects. Thus, each participant did 36 trials (2 scales x 3 pole heights x 3 viewing distances x 2 initial horizontal sizes).

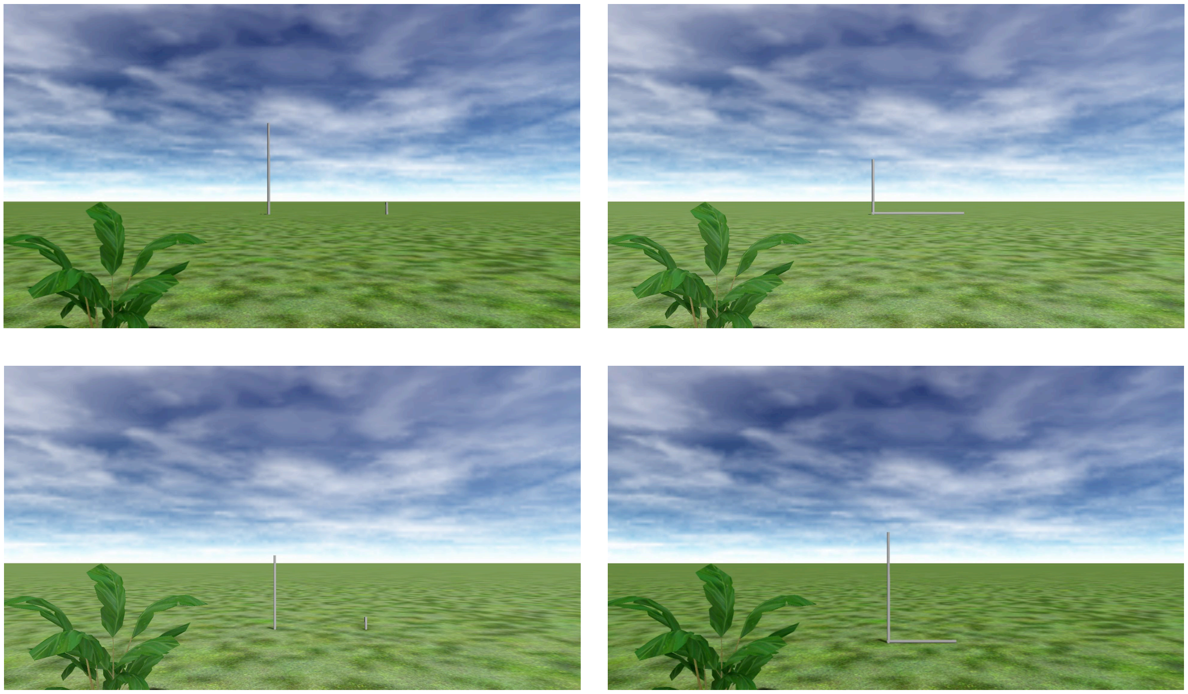

Figure 1. Screenshots of the stimuli and the virtual environment used in Experiment 1. The upper panels show samples of the large-scale versions. In all cases the horizon cuts the poles at eye-height which is shown here as about 1.1 m, appropriate for a seated observer. The original display was stereoscopic, matched to the viewers IPD, and responsive to head movement, with a horizontal FOV of ~90°, providing a rich sense of space and distance.

Results and Discussion

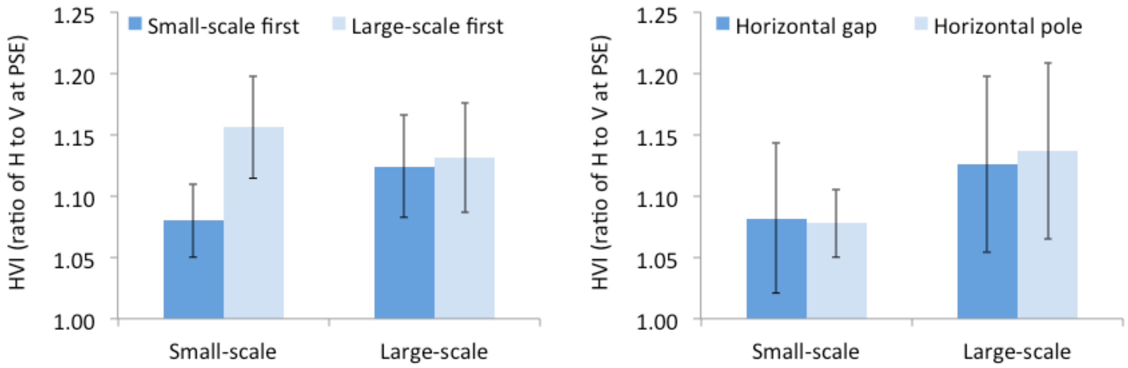

Figure 2. Results of Experiment 1. Left: Mean HVIs and CIs (95%) by object scale and order. Right: First block (between-subject) HVIs and CIs by object scale and type of horizontal extent.

HVI ratios (i.e. the ratio of matched horizontal size divided by actual vertical size) were calculated for each trial at each object scale and collapsed across viewing distance, pole height, and initial position for each participant. A mixed ANOVA with scale order (large-scale first or small-scale first) and horizontal extent type (gap or pole) as between-subject variables and object scale (small, large) as a within-subject variable, showed that there was a reliable interaction between scale and order, $F(1, 32) = 10.4$, $p = .003$, $\eta^2_g = 0.05$. This interaction is depicted in the left panel of Figure 2. To understand this interaction with order, it is necessary to conduct ANOVAs on subsets of the data, split by scale. For the large-scale trials, an ANOVA with order and extent type as between-subject variables, suggested that order and extent type had no effect on the magnitude of the HVI (both Fs < 1). But for the small-scale trials a reliably larger HVI (16%) was found when the large-scale block had been done first than when the small-scale block was first (8%), $F(1, 32) = 9.40$, $p = .004$, $\eta^2_g = 0.23$. (There was still no effect of extent type; $F(1, 32) < 1$.) Moreover, for participants who did the small scale block first, the HVI in the large-scale condition was reliably greater (12%) than that in the small scale condition (8%), $F(1, 16) =$

5.93, $p = .027$, $\eta^2_g = 0.09$, showing that the larger scale produced a larger effect, when the small scale was tested first. The initial comparison of large-scale horizontal and vertical extents apparently influenced the later evaluation of similar smaller-scale extents.

Because order had a complex and unexpected effect on matches, an ANOVA to compare the between-subject small-scale and large-scale effects in the first block of trials was conducted. It showed that the large-scale HVIs were marginally larger (13%) than the small-scale HVIs (8%), $F(1, 32) = 3.71$, $p = .061$, $\eta^2_g = 0.10$, as expected, but there was no difference as a function of the type of horizontal extent used, $F(1, 32) > 1$, as shown in the right panel of Figure 2. These large- and small-scale HVI values are very similar to those reported by Chapanis and Mankin (1967) and by Yang et al. (1999) when comparing large and small outdoor objects. Moreover, there was no effect of extent type, suggesting that a large-scale HVI can be obtained even when an adjustable horizontal pole is compared to the vertical pole.

Experiment 2: Obtaining a large-scale effect with small-scale objects

The order effect in Experiment 1 was not predicted, but it suggests that two different strategies of comparing length may have been available to our participants for the smaller scale objects. Based on the results of Klein et al. (2016), it seems that the large-scale HVI may be a result of using ground-plane based information (such as angular declination) to help estimate *size-at-a-distance* (Gibson, 1950). To test this hypothesis, it is sufficient to separate the horizontal and vertical poles in depth so as to require comparison of size-at-a-distance (accounting for differential distance and foreshortening) rather than of projected size (accounting only for differential foreshortening). It is possible that, for large-scale objects, comparisons based on size-at-a-distance are more salient than projected size, but that projected size comparisons are more salient for smaller objects. If biases in the computation of size-at-a-distance are the appropriate explanation for the large-scale HVI (Klein et al., 2016), simply separating the horizontal and vertical poles in depth ought to fully discourage projected size comparisons.

In addition to this manipulation, we also tested whether manipulating perceived size would be sufficient to induce a ground-plane based evaluation even with objects that were relatively small. To these ends, in Experiment 2, we continued to provide a rich, immersive stereoscopic display of the small-scale scenes and tested the effects of either (1) changing the apparent scaling of the scene by including avatars rendered at 1/5 scale (i.e., to increase the perceived scene scale by a factor of five), or (2) forcing participants to evaluate the small-scale objects as 3D extents (size-at-a-distance) by separating the vertical and horizontal poles in depth. If the smaller HVI in the small-scale case is a result of comparing projected 2D sizes (e.g., via 2D shape analysis), we should find much larger HVIs for small-scale poles when a 2D shape strategy cannot be used or when a 3D strategy is otherwise encouraged.

Methods

Participants. Forty-five undergraduates from Swarthmore College participated in this experiment for payment. All the participants had normal or corrected-to-normal vision. Fifteen participants were run in each of three conditions.

Apparatus. The same apparatus was used as in Experiment 1.

Stimuli and procedure. The virtual environment was mostly identical to the one used in Experiment 1 in the small-scale, horizontal-pole condition except for two changes.

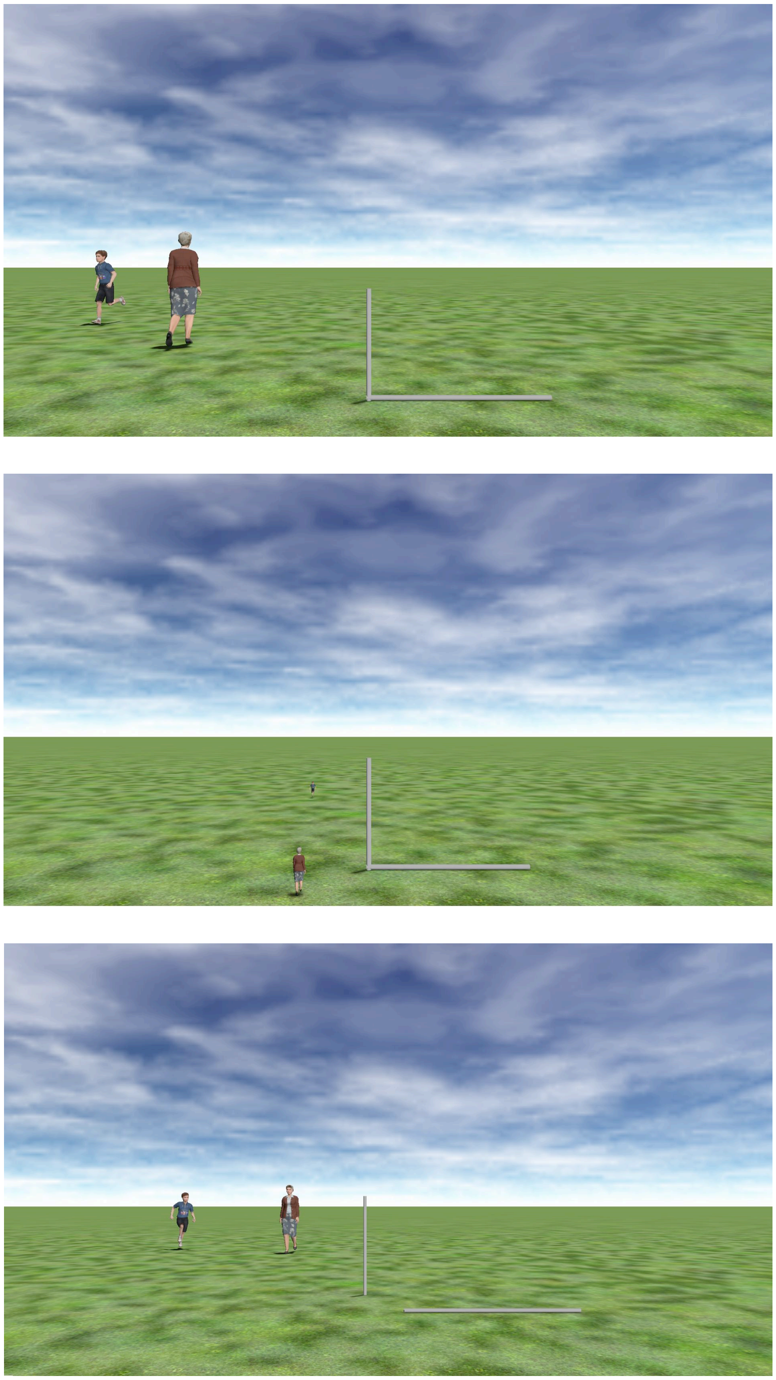

Figure 3. Screen shots representing the three between-subject conditions of Experiment 2. From top to bottom: baseline condition, perceptually-large condition, and depth-separated condition. Note that in the depth-separated condition the vertical pole was closer to the observer than was the horizontal pole on half the trials. The avatars were in continuous motion in the left side of the scene, and the display was immersive (stereoscopic, head-tracked, 90° horizontal FOV).

First, the small foreground plant was eliminated in favor of having animated avatars that locomoted around (one ran, one walked) in the left half of the scene during each trial to provide visual size scaling. These avatars were of normal size in the baseline condition and in the depth-separated condition, but were 1/5 scale in the apparently large-scale (perceptually large) condition.

Second, in the depth-separated condition, rather than appearing at the same distance, the vertical and horizontal poles were always at different distances from the observer. Specifically, as before, the vertical pole could appear at a distance of 4, 5 or 6 m. In each case the horizontal pole would appear at one of the two remaining distances (displaced 0.2 m to the right to avoid any visual intersection with the vertical pole). Thus, this design involved comparisons of horizontal and vertical poles presented at the same 3 distances as in the small-scale conditions of Experiment 1, but with twice as many trials (36) to complete the full factorial design, because of the two different positions of the horizontal pole relative to each vertical pole. Therefore, an equal number of trials were conducted in each of the other conditions (36) by simply running two blocks of 18 trials each for participants in the baseline condition and in the perceptually-large condition. Screen-shots from representative trials in the three conditions are shown in Figure 3.

Manipulation check. At the conclusion of the adjustment trials, the first 20 participants were asked to estimate the height of the tallest pole they had seen in meters or in feet and inches and their judgments were converted to meters for analysis. The mean estimate in the perceptually-large configuration (N=7) was 7.8 m, consistent with the 5x scaling of the 1.5 m pole. This was reliably larger than estimates in both the baseline configuration (N = 7, M = 2.0 m), $t(12) = 2.96$, $p = .012$, and the depth-separated configuration (N = 6, M = 1.9 m), $t(11) = 2.75$, $p = .019$. Thus, at least in memory, the perceived size manipulation successfully changed the apparent scale of the scene.

Results and Discussion

As in Experiment 1, an overall mean HVI was computed for each participant. The mean HVIs are shown in Figure 4, with 95% confidence intervals. An ANOVA by configuration (baseline, depth separated, or perceptually large) confirmed that the HVIs differed by condition, $F(2, 42) = 7.34$, $p = .002$, $\eta^2_g = 0.26$.

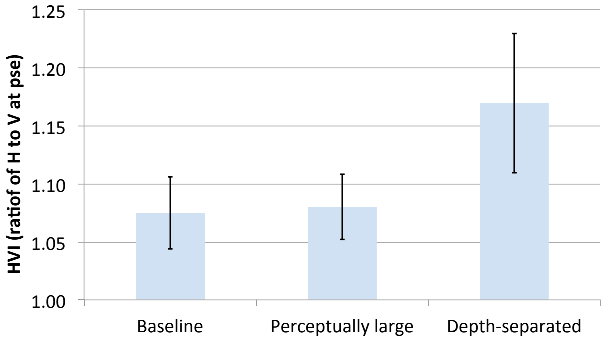

Figure 4. Mean HVIs and CIs (95%) in the three between-subject conditions of Experiment 2.

Our primary hypothesis was that a large-scale HVI would be produced if the two extents were separated in depth. Planned contrasts comparing the depth-separated configuration with the baseline configuration showed that the depth manipulation produced a much larger HVI (M = 17%) than that in the baseline condition (M = 7%), $F(1,42) = 14.6$, $p < .001$, $\eta^2_p = 0.26$. Thus, separating the vertical and horizontal small-scale poles in depth was sufficient to produce a large-scale HVI. This observation is consistent with the hypothesis that greater differential bias is observed when 3D size has to be evaluated.

A second planned contrast indicated that the perceptually-large configuration did not differ from the baseline configuration, $F(1, 42) < 1$. Based on our own experience of the perceptually-large configuration, it seems likely that even though the participants perceived that the objects were large relative to the people in the scene, their view of the scene felt like a giant's view – in which case they did not experience the poles as large relative to themselves.

General Discussion

Large-scale objects show much larger HVIs than do small-scale objects (Chapanis & Mankin, 1967; Dixon & Proffitt, 2002; Klein et al., 2016; Yang et al. 1999). Experiment 1 showed that this was true even when a horizontal pole, rather than a gap, was matched to a vertical pole; essentially all previous studies of the large-scale HVI have compared horizontal gaps to vertical extents (but see Higashiyama, 1996). But the order effect observed in Experiment 1 suggested that the same process of evaluation used for large-scale objects could carry over to small-scale objects as well.

Our hypothesis concerning this process was based on evidence of the importance of the ground plane in defining the large-scale HVI (Klein et al., 2016). Specifically, the comparison of extents in 3D space might be susceptible to biases that are known to affect other kinds of large-scale spatial tasks. By requiring participants to evaluate size-at-a-distance in a simulated 3D space, our depth-separated configuration in Experiment 2 produced a large-scale HVI with small-scale objects.

Thus, although using large-scale objects is sufficient to produce a large HVI, it is not necessary. Rather, it appears that the perceptual system tends to adopt a particular evaluative strategy with objects that are large in scale relative to the observer that can also be induced by simply preventing direct comparison of 2D size. By forcing participants to make estimates of the relative sizes of 3D objects without recourse to projected shape or aspect ratio, we replicate the very large (15-25%) biases observed outdoors for comparisons of large horizontal and vertical extents to egocentric distances (Higashiyama, 1996; Klein et al., 2016; Li, Phillips & Durgin, 2011; Li et al., 2013).

An Angular Model of Bias in 3D Space

The large-scale HVI is but one of many surprisingly large biases in the apparent surface layout of the environment that can be understood in terms of the exaggeration of angular variables in elevation and azimuth (Klein, Li & Durgin, 2016). For example, perceived egocentric distance (the distance between observer and a target on the ground) is linearly compressed whether measured directly (Foley, Ribeiro-Filho, & Da Silva, 2004; Kelly, Loomis, & Beall, 2004) or measured relative to vertical or horizontal extents (Higashiyama & Ueyama, 1988; Jackson & L. K. Cormack, 2007; Li, Phillips, & Durgin, 2011). Moreover, the perceived slope of slanted surfaces (such as hills) is systematically overestimated (Durgin, Li & Hajnal, 2010; Kammann 1967; Li and Durgin, 2009, 2010, 2013; Proffitt, Bhalla, Gossweiler, & Midgett, 1995; Ross 1974).

While all of these phenomena can be described qualitatively in terms of different scaling of the different axes of space (e.g., Wagner, 2006), each has been modeled quantitatively (without free parameters) using data concerning the angular coding of space (e.g., Durgin & Li, 2011; Li and Durgin, 2012). Specifically, according to the scale expansion hypothesis (Durgin & Li, 2011; Durgin et al. 2010; Li & Durgin, 2016), important angular variables, such as angular direction in elevation (upward and downward perceived direction from horizontal), angular direction in azimuth, and optical slant (the angle formed by the line of gaze and the surface where the gaze is directed to), are perceptually exaggerated in the range most relevant for evaluating locomotor space (0° to 50°). The present data concerning the evaluation of perceived frontal extents is similarly quantitatively consistent with the predictions of empirically observed differential angular scale expansion in elevation (1.5) and azimuth (1.25; Li & Durgin, 2016; see also Klein et al., 2016). That is, because angular direction is exaggerated with a gain of 1.5 in elevation but with only a gain of 1.25 in azimuth (Li & Durgin, 2016), the expected perceived ratio of vertical to horizontal for large-scale extents on the ground plane would be about 1.2, which is quite close to the observed ratio for the large-scale HVI.

The present data suggest that the angular expansion hypothesis of the HVI requires a supplementary hypothesis. For large objects the visual system may depend on a size-at-a-distance evaluation strategy that is affected by the large-scale angular biases proposed by the angular expansion hypothesis. However, in the absence of a need to evaluate size-at-a-distance, the visual system may instead assess aspect ratios based on apparent relative sizes corrected only for viewing angle.

Conclusion

The present data provide evidence for a dissociation between the perceptual information available for evaluating the relative sizes of vertical and horizontal lines in a single depth plane and that available for evaluating their relative sizes at different distances (see also Loomis, Philbeck & Zahorik, 2002). We conclude that the ground-plane-based component of the large-scale HVI is distinct from the 2D HVI mechanism (see also Klein et al., 2016). Even the 2D mechanism, however, may involve statistical biases in the relationship between 2D and 3D extents (e.g., Zhu & Ma, 2017) due to the ubiquity of ground planes.

Acknowledgments

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